Debt Overhang, Risk Shifting and Zombie Lending

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Abstract

After bubbles collapse, banks have often rolled-over debt at subsidized rates to insolvent borrowers or "zombie firms." This paper explores the incentives to restructure debt in a game with risk shifting under debt overhang. We provide conditions under which it is privately optimal to zombie-lend even when it is socially inefficient. When a firm becomes insolvent, the firm loses access to competitive funding and its bank can exert monopoly power. The bank prefers to zombie-lend given that flowing funds for investment is not profitable due to risk shifting and liquidation entails costs. The model explains the inefficiency of traditional policies in the presence of zombies such as bank recapitalization and monetary policy and highlights the necessity of debt haircuts.

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1 Introduction

Sharp drops in asset prices can wipe out collateral value and spell trouble for both banks and firms. Insolvent firms in a situation of debt overhang may continue to operate if banks roll over their debt at subsidized rates, a practice known as zombie-lending. These "zombie firms" are kept afloat but fail to grow. This phenomenon has substantial negative effects on investment and productivity. In the face of debt overhang, decentralized bargaining should theoretically lead to a welfare improving situation, leading to a liquidated firm or to debt restructuring. However, the experiences of Japan during the 1990s and of Europe during the 2010s show that there can be substantial delays in this recovery (see Figure 1).

This paper develops a model of bank-firm interaction to explain how zombie firms arise in decentralized equilibrium and continue to thrive, even when restructuring is formally allowed. Zombie-lending is defined in the paper as the practice of continuously lending for survival to an insolvent firm, the zombie. Previous studies of debt forbearance have focused on regulatory forbearance of a troubled bank or on informational frictions (See Related Literature). The model generates the phenomenon in a full information setting where firms have access to solvent banks and profitable projects. Policy implications are vastly different given that previous studies assume that firms are unprofitable and therefore restoring efficiency requires the liquidation of their assets.

The result of zombie lending as an equilibrium strategy emerges from the disconnect between the incentives of the bank and the firm. When debt is excessively high, the firm cannot access the competitive credit market and is thus locked-in with the incumbent bank. The bank can then extract a large surplus from the captive firm without funding any investment. Alternatively, if the bank would restructure and extend a loan to enact an efficient project, the required repayment to maximize profits would need to be large. The firm would then, overburdened, shift the risk of the investment project. Anticipating the firm's potential "gamble," the expected return of extending fresh funds is low. Therefore, the bank prefers to take the firm's business-as-usual revenue stream for itself. In other words, the bank takes the firm's profits, ignoring previous debt services. Thus, it subsidizes the firm's debt payments, turning the firm into a zombie.

The model presents a game among three agents, a firm and two banks, that lasts three periods. The firm has a revenue stream with its "business as usual" technology and an exogenous level of debt contracted with one of the banks, the incumbent. The firm can invest in one project chosen from a risk continuum. If successful, the project allows the firm to increase its revenue stream permanently. When initial debt is low, i.e. "normal times," the firm can repay its incumbent bank and borrow from either of the two banks to finance investment. Bank competition pushes the cost of borrowing until banks make zero profits. The firm invests in a project with socially optimal risk and social welfare is maximized.

However, when the firm suddenly becomes insolvent, a hold-up problem arises. During "crisis times," the firm is no longer able to access the competitive market because the second bank would never find it profitable to lend to an insolvent company. Thus, the incumbent bank becomes a monopolist and, as such, has two broad strategies in the first period: it can $(i.)$ liquidate the firm early and put the proceeds in a risk-free technology, or $(ii.)$ extend credit, be it via restructuring or survival lending. The firm, in turn, can use the funds at its own discretion in the intermediate period. At the end of the game, the bank is either repaid or liquidates the firm. In this model, we demonstrate that there are situations under which the bank decides to keep the firm under survival lending even without hopes of "resurrection." Importantly, the bank can zombie-lend even when the firm has a profitable project or when it is socially beneficial to restructure debt.

Section 2.5 extends the model to the case of a financially distressed bank and firm simultaneously. In this case, the bank's assets are subject to a shock that may render it bankrupt. If the bank decides to liquidate the firm early, it must acknowledge the loss and will thus be in a more fragile position to face the shock. A bank in a strong capital condition can bear the loss and liquidate the firm, although this is socially inefficient when the firm possesses profitable projects. In contrast, a weak bank, fearing the shock, may not be willing to do so and would keep the firm as a zombie.

The analysis shows that the conditions for zombie-lending and the effects of policy depend on whether the firm has positive operational profits or not. Therefore, the firm's ability to profit independently of its financial burden impacts the outcomes of policy interventions. A high scope for risk shifting and large disruption costs are necessary for zombie-lending when there is debt overhang. On the other hand, firms with operational losses will become zombies only if banks are financially distressed, as otherwise they would be liquidated.

The model yields several policy prescriptions. First, partial debt forgiveness in the form of a debt haircut increases welfare but is not privately optimal. The bank is unwilling to privately reduce the debt burden because the business-as-usual income stream from the firm is, given the possibility of risk shifting, higher than the expected return under the renegotiated plan. A haircut on debt decreases the bank's market power, allowing the zombie-firm to regain access to the competitive market and invest. Social surplus is augmented and redistribution can improve the bank's welfare as well. Second, recapitalization is inefficient to increase investment. Recapitalizing a bank changes its incentives but does not change the firm's. A more capitalized bank may decide to liquidate a viable firm (i.e., one with a profitable project and positive present value) but will not lead to higher investment. The optimal policy response to zombie-lending includes simultaneous bank recapitalization and a haircut on firm debt. Third, when dealing with zombie firms, monetary policy is ineffective under certain conditions. Decreasing the interest rate decreases the opportunity cost for the bank, but not the firm's effective rate, due to the underlying incentive problem.[1]

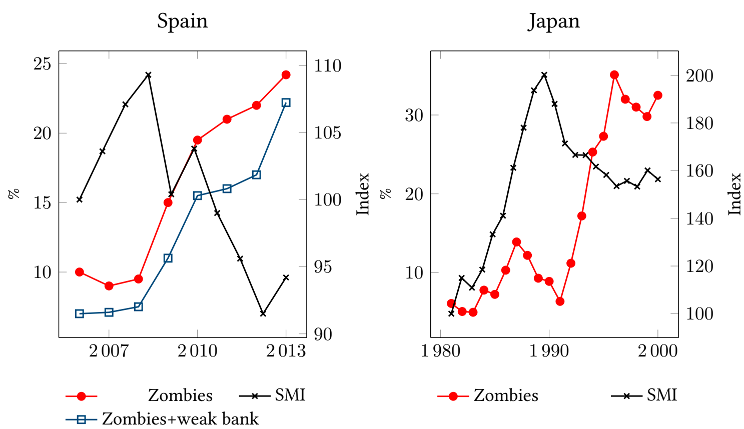

Figure 1: Percentage of Zombie Firms (left axis) and Stock Market Index (SMI, right axis). Zombies receive below the market rates. Weak banks have a risk-based capital below 1 percentage point above the required capital. Sources Aragon [2018] and Caballero et al. [2008].

Related Literature. Zombie firms were first identified after the burst of the Japanese asset price bubble, and the country's growth has remained stagnant for decades since. Similar dynamics were observed after the European debt crisis, and several facts are common to both episodes. First, firms receiving zombie-lending are highly indebted, and their prevalence is higher after the burst of a bubble [Caballero et al., 2008, Hoshi and Kashyap, 2004, Andrews and Petroulakis, 2019, Aragon, 2018]. Second, these firms are more likely to be in a lending relationship with a financially distressed bank [Peek and Rosengren, 2005], particularly when restructuring is costly [Andrews and Petroulakis, 2019]; see Figure 1. Third, these firms are less likely to invest [Peek and Rosengren, 2005, Kalemli-Ozcan et al.,

[1]Other alternatives for the resolution of debt overhang can be understood within the model. See Section 4.

2018]. Finally, when zombies are widespread, recapitalizing banks and lowering the interest rate have had mild or nil effects on reigniting the economy [Schivardi et al., 2017, Acharya et al., 2016, Paligorova and Santos, 2017, Berger et al., 2016]. This paper relates to the literature by showing how this phenomenon can arise in a decentralized equilibrium and accounts for these regularities by focusing on a double decked incentive problem.

Zombie-lending is, in essence, the study of debt forbearance, and, as such, lies at the intersection between the literature of debt overhang and renegotiation. The debt overhang literature starting with Myers [1977] highlights that large amounts of debt lead to under-investment. Such work spans a large literature focusing on a variety of issues mostly focusing on the incentives of borrowers. This paper, instead, focuses on the incentives of both borrowers and lenders. Kovrijnykh and Szentes [2007] focus on both actors as they study countries in debt overhang. We simplify their setup and apply it to firms which, unlike countries, can be liquidated and have a scope for shifting risk. These two differences substantially change the space of strategies and outcomes.

The literature on renegotiation[2] highlights that, when there is a bargaining surplus, default is inefficient and renegotiation mitigates the under-investment result arising from debt overhang. As such, the theory presented in this paper relates to delays in this process. Admati and Perry [1987], Vilanova [2004], and Kahl [2002] explain these delays based on information frictions and relative bargaining powers between borrowers and lenders. In our model, shocks are permanent and the evolution of the firm's output is known by both parties. The delay here arises because the bank does not find it profitable to renegotiate at any point, as it can take the totality of the firm's business-as-usual profits, and if it did decide to lend further funds, the firm would increase the risk, thus rendering it unprofitable.

The literature on debt forbearance has usually focused on the incentives of the regulators [Hellwig et al., 2012, Bruche and Llobet, 2013, Aghion et al., 1999] or, in the case of state owned companies, the government [Berglof and Roland, 1998, Aghion and Bolton, 1992]. The literature focuses on regulatory schemes where the bank can hide a bad loan from their principal. This paper, instead, abstracts from the incentives of the regulators to focus solely on profit-seeking agents to show how competitive markets can lead to the same situation, and goes to explore macroeconomic policy under this condition. We show that regulatory capture is not necessary for the existence of these firms. An exception focusing on forbearance from the point of view of the lenders is Rajan [1994]. In that paper, a banker

[2] See for example Hart and Moore [1998], Hellwig [1977], Hart and Tirole [1988], Frantz and Instefjord [2019], Favara et al. [2017], Pawlina [2010].

with reputational concerns decides to "gamble for resurrection" when faced with an unprofitable firm. Hu and Varas [2021] provide a theory of zombie lending and is closest to this paper in terms of the object of study. Their theory does not rely on capital requirements, but instead on information production, information acquisition, and reputation formation. An entrepreneur with a good reputation will be able to roll over loans even when insolvent. The theory put forward in this paper complements this explanation by focusing on debt overhang and monopolistic power under full information. This alternative explanation leads to different policy implications. Whereas explanations based on informational frictions normally imply a constrained efficient solution, under the theory in this paper, moral hazard and risk shifting interact to prevent the renegotiation outcome from arising. This has crucial implications for policy. Regulators should not put effort into persuading banks to disclose non-viable loans so that they liquidate the firms, but rather into breaking up the debt overhang problem.

Section 2 presents the model. The case of an undercapitalized bank is in Section 2.5. Policy is discussed in Section 3. Section 4 concludes and discusses results and assumptions.

2 The Model

This paper presents a simple game that provides theoretical insights on the behavior of borrowers and lenders when risk shifting is possible in the investment technology. The firm has an outstanding level of debt with one of the banks, "the incumbent." This debt may receive an unexpected shock that leads to debt overhang. The model shows that, in this situation, the decentralized renegotiation equilibrium may be inefficient. In this equilibrium, firms endogenously fail to invest even with available profitable opportunities. Finally, we explore different policies and show how they can implement an efficient allocation.

2.1 Environment

There are three risk-neutral agents: a firm and two banks. They live for three periods: 0, 1, 2. The firm seeks resources to finance a risky project in a lending market and has an inherited debt with one of the banks, the incumbent. In period 0, banks compete *á la* Bertrand to provide funds to the firm. An offer consists of a payment in period 1 and a payment in period 2. A bank can also offer to liquidate the firm in period 1 (and put the proceeds in a risk-free technology) or in period 2. The firm may choose to accept or reject these offers.

The firm possesses a technology to produce output. In period 1, the firm produces using its traditional technology. It may also decide to carry on a risky investment project. The project, if successful, improves the technology which the firm can then use in period 2 to produce and pay back debt. If the firm fails in period 2, the firm is liquidated and its assets are sold. The timing of the game is shown in Figure 2.

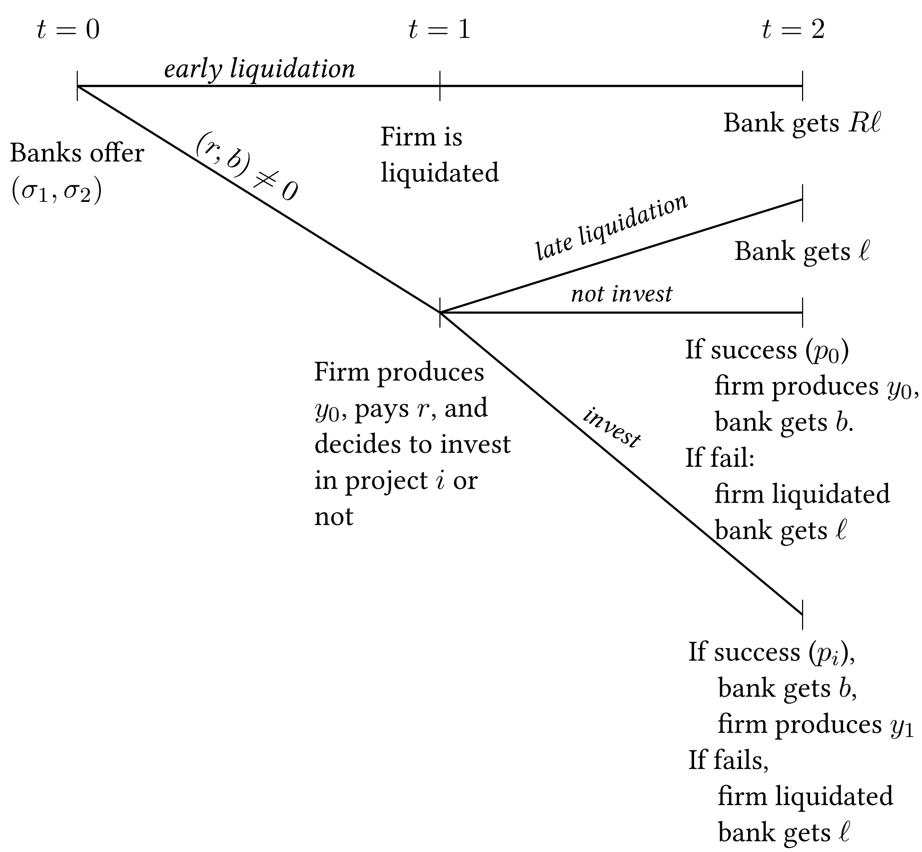

Figure 2: Timing of the model

Banks. There are two banks: the incumbent bank and the competitor bank. The incumbent bank has a previously contracted claim on the firm, D_0. In period 0, the banks will make offers to the firm. Each bank's strategies are denoted as a pair for the actions in period 1 and 2 respectively as (σ_1, σ_2). In period 0, each bank can make two general types of offers for each period: flowing funds or liquidation. Importantly, flowing funds includes, as a special case, the restructuring of the original debt.

The banks can offer early liquidation (in period 1) or late liquidation (in period 2). If the bank decides on early liquidation, it will obtain the value ℓ and can put the proceedings in a risk-free technology that pays $R > 1$ per unit. Under this strategy, the bank will receive zero from the firm in period 2 as the firm no longer exists, but its proceeds will be $R\ell$. In this case, the bank's strategy will be $(\sigma^\ell, 0)$, where σ^ℓ implies that the bank liquidates the firm. Clearly, no firm that can repay its original debt would choose to accept this offer from its incumbent bank. Moreover, if the competitor bank, with no claims on the firm, were to make this offer, it would never be accepted. Therefore, this strategy can only be used by the incumbent bank when the firm is insolvent.

A bank that decides not to liquidate the firm at all can offer a pair (r, b), composed of a payment r in period 1 and a payment of b in period 2. If the firm cannot repay in period 2, it will be liquidated. It is important to clarify that r and b may be greater than zero (in which case the bank gets money from the firm) or less than zero (in which case the bank extends new funds). Moreover, (r, b) can include the strategy of restructuring on D_0, which is a sunk cost. Finally, (r, b) includes the case of subsidized lending when r is smaller than the contracted repayment on D_0.

There is also an intermediate strategy where the bank chooses late liquidation and sells the assets at $t = 2$ to a different entrepreneur. In this case, the bank may choose to extend or receive funds from the firm at $t = 1$. We denote these funds as r^P. This intermediate strategy is thus (r^P, σ^ℓ).

The incumbent bank's expected profits, $E\Pi$, for each of its strategies is given by

$$E\Pi(\sigma_1, \sigma_2) = \begin{cases} R\ell & \text{if } (\sigma_1, \sigma_2) = (\sigma^\ell, 0) \\ r^P + \ell & \text{if } (\sigma_1, \sigma_2) = (r^P, \sigma^\ell) \\ p(b)b + (1 - p(b))\ell + Rr & \text{if } (\sigma_1, \sigma_2) = (r, b) \end{cases}, \qquad (1)$$

conditional on the contract being accepted by the firm. In Equation 1, the liquidation value of the firm is $\ell = \max\{\phi E\pi, 0\}$, where $E\pi$ is the net present value of the firm under competitive markets and $(1 - \phi)$ is the disruption cost. Notice that if the firm's profits are negative, the liquidation value is zero. When the bank decides to do an early liquidation of the firm, it will receive $R\ell$, due to earnings on the risk-less technology. If the bank chooses late liquidation, it will only receive ℓ from the liquidation, but will also earn r^P from the first period. Finally, if the bank decides to extend new funds, it will get r in the first period, which will go into the risk-less technology earning Rr, and b in the second period, conditional on the firm being successful, which occurs with probability $p(b)$. If the project is unsuccessful, the bank will get its liquidation value, ℓ. The bank understands that different repayments b will provide different incentives for the firm, which translate into different success probabilities, $p(b)$.

The competitor bank's potential strategies are denoted by primes: (σ_1', σ_2'). The potential strategies are the same as the incumbent bank: extend loans for both periods, (r', b'), immediate liquidation, $({\sigma'}^{\ell}, 0)$ or extend funds and late liquidation, $(r^{P\prime}, {\sigma'}^{\ell})$. The profit schedule for the bank, $E\Pi'(\sigma_1', \sigma_2')$, is thus the same as the incumbent bank, although they will face different incentive constraints. Thus, if the competitor bank offers contracts (σ_1', σ_2'), its profits are

$$E\Pi'(\sigma_1', \sigma_2') = \begin{cases} R\ell & \text{if } (\sigma_1', \sigma_2') = ({\sigma'}^{\ell}, 0) \\ r^{P\prime} + \ell & \text{if } (\sigma_1', \sigma_2') = (r^{P\prime}, {\sigma'}^{\ell}) \\ p(b')b' + (1 - p(b'))\ell + Rr' & \text{if } (\sigma_1', \sigma_2') = (r', b') \end{cases}, \quad (2)$$

conditional on the contract being accepted. Notice that, unlike the incumbent bank, the competitor bank has no original claims on the firm, D_0. Therefore, even if it can theoretically offer to liquidate the firm, this is a contract that will never be offered in equilibrium.

Firms. At the beginning of the game, the firm has a technology that allows it to obtain a revenue of $y_0 = p_0^{-\alpha}$, where p_0 is the probability of success or baseline risk. The parameter $\alpha \in [0, 1]$ captures the elasticity of revenue to the riskiness of the project and measures the scope for risk shifting: the safer the project, the lower the revenue in case of success. In period 1, the firm can produce using this inherited technology. In period 2, the expected revenue with this technology is $Ey_0 = p_0 y_0 = p_0^{1-\alpha}$ and, if it fails with probability $(1 - p_0)$, the firm gets zero. In other words, y_0 is the "business-as-usual" revenue stream.

In period 1, besides producing using its traditional technology (and getting y_0), the firm can also invest in one project that improves technology. Projects are chosen from a continuum and the probability of success for each, p_i, ranges from $[0, 1]$. Thus, p_i close to one implies a safe project and p_i close to zero implies a risky project. In case of success, each project induces an increase in revenue from $y_0 \equiv p_0^{-\alpha}$ to $y_1 = p_i^{-\alpha}$ in period 2. The cost of investment, X, is constant and the same for all projects p_i. Finally, the choice of the project is not contractible: the firm cannot commit to invest along a particular risk profile.

The firm has an exogenous, previously contracted level of debt, D_0, with the incumbent bank. This is a sunk cost. If the firm refuses to pay the contracted debt with its incumbent, it can be liquidated and its assets seized.

Thus, a firm that is not liquidated will choose whether to invest or not by maximizing the value of the firm at the beginning of the game,

$$\max_{\mathbb{1}_I} E\pi = \max_{\mathbb{1}_I} \{y_0 - r - \mathbb{1}_I X + [\mathbb{1}_I EV_i(b) + (1 - \mathbb{1}_I)p_0(y_0 - b)]\} \quad (3)$$

Here y_0 is the level of output in period 1, $\mathbb{1}_I$ is an indicator function equal to one if the firm invests and zero if it does not, r is the payment in period 1 to the bank, which may be positive (in which case the bank takes funds) or negative (in which case the bank extends funds). The term in square brackets represents profits in period 2 if the firm invests ($\mathbb{1}_I = 1$) or if it does not invest ($\mathbb{1}_I = 0$). Notice that r may be contracted with a different bank than the one that loaned D_0. The value of the firm in period 2, if it decides to carry on project i, is

$$EV_i(b) = \max_i \left\{p_i(p_i^{-\alpha} - b)\right\},$$

where p_i is the probability of success of project i and b is repayment on new funds. The firm can choose to carry on a new project or remain in its business-as-usual mode, obtaining an expected revenue of $p_0 y_0$. Note that EV_i has an inverse-U shape with a maximum at an intermediate efficient level of risk-taking. The first order condition, conditional on the firm investing, is

$$p^*(b) = \left(\frac{1-\alpha}{b}\right)^{1/\alpha}, \quad (4)$$

which depends inversely on b. Equation (4) shows how the firm's attitudes towards risk are shaped by the level of repayment expected by its bank. The reaction function presents risk shifting: the higher the debt, the lower the risk of the project chosen in equilibrium.

Constraints. Banks offer contracts and firms decide whether to accept or reject them in order to maximize profits. There are several constraints that should be considered.

(A) Incentive Compatibility for firm

First, the firm will accept any contract with weakly positive profits, since otherwise it would be liquidated, in which case it gets zero. Thus, the value of the firm should satisfy

$$\max_{\mathbb{1}_I} \{y_0 - r + \mathbb{1}_I(EV_i(b) - X) + (1 - \mathbb{1}_I)p_0(y_0 - b)\} \geq 0 \tag{5}$$

In other words, if the firm is offered (r, b), it will only operate if profits are weakly positive, and will optimize its strategies in reaction to the offers. If the firm is liquidated, it will get a value of zero, which is its outside option.

(B) Feasibility

Secondly, the firm needs to be able to pay for the contracts. In other words, contracts need to be feasible. In period 1, this includes the payments for investment, X, and to the bank, r. This also can be interpreted as feasibility of investment. In period 2, this takes into account the repayment to the bank. If the firm decides to invest, its profits will be y_1, whereas if the firm stays with its business-as-usual project, it will get y_0. This entails the following set of constraints:

$$\begin{cases} y_0 - r - \mathbb{1}_I X \geq 0 & t = 1 \\ y_1 - b \geq 0 & t = 2 \wedge \mathbb{1}_I = 1 \\ y_0 - b \geq 0 & t = 2 \wedge \mathbb{1}_I = 0 \end{cases} \tag{6}$$

(C) Incentive Compatibility for Competitor Bank

Third, following Myers [1977] and Fama and Miller [1972], we assume debt seniority in repayment.

Assumption 2.1. *Old debt is senior to new debt*

Assumption 2.1, is crucial for the results of this paper. If we did not make this assumption, the issuing of new claims would dilute the expected repayment for the incumbent bank. If the firm becomes insolvent, the incumbent bank would have to share the recovery value with the competitor. Anticipating this behavior, banks would refuse to lend in the first place. An equivalent statement is to say that the competitor bank will make an offer (r', b') only if the firm can make enough profits as to pay its original debt, D_0 [Kovrijnykh and Szentes, 2007]. The following constraint describes these contracts,

$$\max_{\mathbb{1}_I} \left\{y_0 - r' + \mathbb{1}_I(EV_i(b') - X) + (1 - \mathbb{1}_I)p_0(y_0 - b')\right\} \geq D_0 \tag{7}$$

(D) Incentive Compatibility for Incumbent Bank

The incumbent bank will offer a contract (r, b) such that the expected repayment is at least as high as the opportunity cost of all the extended funds in the first period. If the competitor offers a contract that satisfies this constraint, it is also profitable for the incumbent bank to do the same. Without loss of generality, it is assumed that the incumbent is the one accepted when both banks make the same offer. Therefore, the following incentive compatibility will only apply in equilibrium to the incumbent bank when it decides to extend new funds for investment,

$$p(b)b + (1 - p(b))\ell \geq Rr \ , \tag{8}$$

where $p(b)$ is the success of the project given by Equation (4) and ℓ is the liquidation value of the firm, which the bank will obtain in case the project fails. Rr is the opportunity cost of extending new funds.

2.2 Equilibrium Analysis: Market Structure

Market structure depends on the inherited level of debt. For low levels of debt, the firm is solvent and the competitor bank is willing to make offers that push the incumbent to the competitive outcome. Both banks will make a competitive offer to finance investment. The incumbent bank will make an additional offer in which it requests the original debt to be repaid. On the other hand, if the inherited level of debt is large enough such that the firm is insolvent, then the incumbent bank becomes a monopolist. This is a situation of debt overhang, and, in this case, the incumbent bank will maximize profits subject to the incentive constraints of the firm. The firm will choose a project from its full array of possibilities. This is summarized in Proposition 2.2.

Proposition 2.2. (Market Structure) *Assume the firm possesses a project that increases social surplus and increases its expected output. Then, there exists a threshold for the inherited level of debt, $\bar{D}(y_0)$, such that*

- *(Competitive markets) For $D_0 < \bar{D}(y_0)$, both banks make offer $\hat{r} = -X$ and $\hat{b} = RX$. The incumbent bank makes an additional offer of $\hat{\hat{r}} = D_0$.*
- *(Debt overhang) For $D_0 \geq \bar{D}(y_0) > y_0$, the incumbent bank maximizes Equation (1), subject to Equation (6), Equation (8), and Equation (4) if it extends new funds.*

The threshold $\bar{D}(y_0)$ is a function of y_0, meaning that the threshold depends on the level of debt with respect to business-as-usual profits. In other words, in order to generate debt overhang it is equivalent to shock the original level of output (decreasing it) or the original level of debt (increasing it).

In the next sections, we analyze the properties of the equilibrium under "normal times" and "crisis times." The difference between the two is that an unexpected shock pushes the original debt above the threshold. This isolates ex-ante precautionary motives. The assumption of zero-measure shock is further discussed in the conclusions.

2.3 Normal Times: Equilibrium under Competition

When the firm has access to competitive lending, competition will push the banks' profits towards zero. Both banks will offer $b = RX$ in period 2 and to extend funds $-r = X$ in period one; the incumbent bank will also get D_0. In this section, we normalize D_0 to zero purely to ease exposition, as it is sunk and does not distort any decision. Without loss of generality, assume that the firm will choose its incumbent bank when presented with equal offers from both. The following proposition summarizes the reaction of the firm under competitive markets.

Proposition 2.3. *Under competitive markets, as in Proposition 2.2, and $D_0 \leq \bar{D}(y_0)$, the firm will implement project $\hat{p}$, given by*

$$\hat{p} = p(RX) = \left(\frac{1-\alpha}{RX}\right)^{1/\alpha} \tag{9}$$

for $RX \geq 1 - \alpha$.

In other words, risk-neutral banks will compete in prices to provide loans until they make an offer at the opportunity cost of those funds. The firm will carry on the project given that it is preferable to the revenue stream from its business-as-usual technology. In doing so, it will take on an efficient level of risk, given by the reaction function of the firm. Firms repay their debt (normalized to zero), and request funds to do an investment project. They receive the competitive rate and take on an optimal level of risk.

This paper focuses on investment projects that are efficient. In this paper, a project is efficient if it is voluntarily carried out under competitive markets and increases social surplus. The following proposition summarizes the set of investment projects for which this is true.

Proposition 2.4. *If* $RX \in I_X \equiv \left[(1-\alpha), \max\left\{\ell, (\alpha p_0^{\alpha})^{\frac{\alpha}{1-\alpha}}\right\}\right]$, *the project is efficient.*

The lower bound of the set I_X ensures that probabilities are bounded by 1. The upper bound ensure that, under perfect competition, firms prefer the project over their business-as-usual revenue and that social surplus is increased. Thus, for all $RX \in I_X$, under competitive markets, social welfare is increased and a risky project is enacted, according to Equation (9).

2.4 Crisis Times: Equilibrium under Monopolistic Banks

Now suppose there is an unexpected shock to original debt, ξ, such that $D_0' = D_0 + \xi \geq \bar{D}$. Following Proposition 2.2, the bank will have monopoly power given that the firm is now insolvent. Note that a shock to $\bar{D}$ is equivalent to an output shock, $\epsilon \in [0,1]$, to y_0, such that $y_0' = \epsilon y_0 = \epsilon p_0^{-\alpha}$. The firm is financially distressed (i.e., insolvent due to financial obligations) but still has positive operational revenues. The case of a firm with operational losses is studied in the following subsection.

According to Proposition 2.2, the bank now faces a maximization problem subject to certain constraints. In this case, it acts as a monopolist and can extract more than the competitive rate. The bank's broad strategies are to $(i.)$ extend funds (r,b) or $(ii.)$ liquidate the firm (either early, in which case it gets 0 from the firm in the second period, or late, in which case it can charge r^P in the first period). In other words, possible strategies for the incumbent bank, (σ_1,σ_2), are $\left\{(r,b), (\sigma^{\ell},0), (r^P,\sigma^{\ell})\right\}$.

Two remarks are in order. First, initial debt D_0' is a sunk cost and will play no role beyond granting monopoly power. Second, even though (r,b) is a continuous decision, there will only be two cases to consider for the initial repayment: r will be either all operational profits from the firm ($r = \epsilon y_0$) or will fund investment ($r = \epsilon y_0 - X$), because otherwise there are idle funds or not enough to invest, both of which are sub-optimal. The set of possible equilibrium strategies is described in Proposition 2.5.

Proposition 2.5. *Under debt overhang as in Proposition 2.2, The set of the bank's equilibrium strategies* (σ_1,σ_2) *belongs to the set* $\Sigma = \{I, Z, L, ZP\}$, *where*

1. *(I) Funding investment, which sets* $(r^I = \epsilon y_0 - X, b^I = \frac{\ell}{1-\alpha})$,
2. *(Z) Zombie-lending, which sets* $(r^Z = \epsilon y_0, b^Z = \epsilon y_0)$

3. *(L) Liquidating, which sets* $(\sigma^\ell, 0)$.
4. *(ZP) Partial Zombie-Lending, which sets* $(r^P = \epsilon y_0, \sigma^\ell)$.

In strategy (Z), the bank takes the full operational profits from the firm without lending for further investment. We interpret (Z) as zombie-lending. Notice that even if $r \geq 0$, this implies subsidized lending even if revenue is positive. This is because, when taking into account the roll-over costs, the firm's profits are indeed negative. The original debt, D_0, is a sunk cost at this point but made the firm insolvent. Thus, given that $D_0' \equiv \xi \geq r^Z = \epsilon y_0$ as per Proposition 2.2, the firm is indeed receiving subsidized lending. In strategy (ZP), the bank offers $(r^P, \sigma^\ell = 1)$. We interpret (ZP) as partial zombie-lending. In this case, the bank obtains the full operational profits of the firm in the first period but liquidates the firm in the second. Finally, notice that strategy (I) includes the efficient restructuring outcome, as the bank can reorganize previously contracted debt between the two periods, as well as funding if there is an efficient project available.

Figure 3 shows expected profits for the various strategies available to the bank. The bank's profit schedule under strategy (I), $E\Pi^I$, is non-monotonous. This arises because the possibility of risk shifting decreases the firm's incentives to enact the efficient project. Larger repayments, b, increase risk and thus decrease expected repayment. $\underline{b}$ is the minimum possible repayment, following from the bounds of I_X. The bank's profit schedule under strategy (Z) is given by $E\Pi^Z$. Given that the firm does not change its technology, there is no scope for risk shifting, and the probability of success remains p_0. Profits increase monotonically until truncated at the point of zero profits for the firm, i.e., the maximum possible repayment. The profit schedule from liquidation, $E\Pi^L$, is constant at $R\ell$. The partial zombie-lending strategy, (ZP), also yields constant profits, $E\Pi^{ZP}$. The bank decides to take the firm's revenues as payment in the first period and puts them in the risk-less technology. In the second period it liquidates the firm. Thus, it foregoes the interest on liquidation, as it occurs in period 2.

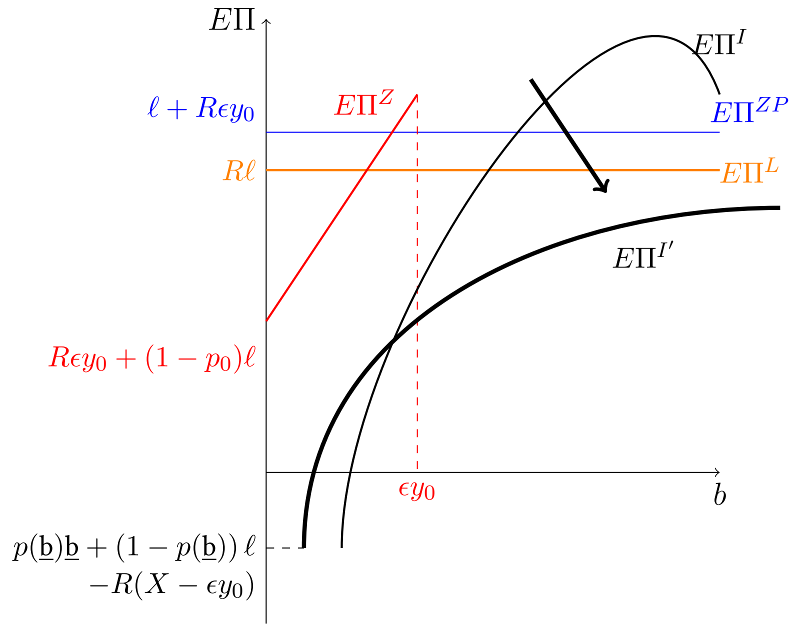

Figure 3: Profit functions for each strategy of the bank. The arrow shows the flattening effect on $E\Pi^I$ as α increases.

Formally, the bank solves the following maximization problem,

$$\max_{\{Z,I,L,ZP\}\in\Sigma} \left\{E\Pi^I, E\Pi^Z, E\Pi^L, E\Pi^{ZP}\right\} \quad \text{s.t}$$
$$(i) \quad E\Pi^Z(r^Z, b^Z) = p_0 b^Z + (1-p_0)\ell + Rr^Z \geq 0$$
$$(ii) \quad E\Pi^L(\sigma^\ell, 0) = R\ell \geq 0$$
$$(iii) \quad E\Pi^{ZP}(r^P, \sigma^\ell) = Rr^P + \ell \geq 0$$
$$(iv) \quad E\Pi^I(r^I, b^I) = p(b^I)b^I + (1-p(b^I))\ell + Rr^I \geq 0$$
$$\text{s.t} \quad p(b) = \left(\frac{1-\alpha}{b}\right)^{1/\alpha}$$
$$(v) \quad \text{Feasibility (Eq. (6)).}$$

The only constraint missing is the incentive compatibility for the incumbent bank. Notice, however, that this is slack. In strategy (I), if $r \leq 0$ and $b \geq 0$, incentive compatibility holds trivially. If $r \geq 0$, as in strategy (Z), then it is slack due to feasibility. For strategy (ZP), it is implied by the maximization between $E\Pi^{ZP}$ and $E\Pi^{L}$.

When there is a debt overhang problem, the amount of money that the bank can obtain from the firm is not determined by the opportunity cost. Rather, the bank maximizes the amount of money that can be extracted from a captive firm. Therefore, the bank can push the debt beyond the competitive level of debt. The higher repayment pushes the firm towards higher risk-taking, thus lowering social welfare. The following result follows,

Lemma 2.6. *When firms are in debt overhang as in Proposition 2.2, $RX \in I_X$, and $\ell \geq (1-\alpha)^2$, social welfare is not higher than under competitive markets. Liquidation is socially inefficient.*

This Lemma states that, under debt overhang and in the presence of efficient projects, social welfare is lower. The restriction of $\ell \geq (1-\alpha)^2$ ensures that probability is well defined. Welfare is lower due to an unnecessary level of risk-taking, unexploited investment opportunities, or inefficient liquidation. Liquidation is inefficient because the firm is financially distressed, but its operational revenues are still positive.

A financially insolvent firm with a profitable project does not receive fresh funds, and instead obtains help rolling over their original debt. The key is that the firm is in financial distress but still viable: its revenues are positive when not taking debt payments into account. The lack of solvency is ensured under debt overhang. We now find conditions under which the zombie-lending strategy will be chosen by the bank, even when there are profitable projects, and renegotiation and liquidation are allowed as strategies.

Theorem 2.7. (Zombie Firms) *Given parameters $(p_0, \epsilon, \phi, \alpha, RX) \in [0,1]^4 \times I_X$, under debt overhang as in Proposition 2.2, when $\alpha \to 1$,*

- *If $\epsilon \leq p_0\left(\frac{R-1}{R-\phi R+\phi}\right)$, bank chooses (L)*
- *if $\epsilon \in [p_0\left(\frac{R-1}{R-\phi R+\phi}\right), p_0\left(\frac{\phi}{1-\phi}\right)]$, bank chooses (ZP)*
- *if $\epsilon \geq p_0\left(\frac{\phi}{1-\phi}\right)$, bank chooses (Z).*

Theorem 2.7 states that, when there is a high degree of risk shifting, the bank may find it more profitable to keep the firm alive by rolling over its financial costs without investing even if the firm has a profitable project.

When the shock to output shock is low (i.e., high ϵ) the bank will do pure zombie-lending, (Z). This is because the profits to be extracted without new projects in the firm are large enough. In particular, it is large with respect to the disruption cost, $1 - \phi$. For intermediate levels of ϕ, the bank will choose partial zombie-lending. This means the bank will prefer to keep the firm alive during $t = 1$ but will then liquidate it at $t = 2$. This relates to the interest rate, as higher interest rates are a force in favor of early liquidation. Higher values of ϕ and lower values of p_0 decrease the threshold for ϵ, thus enlarging the parameter space for zombie lending to arise. Low disruption costs (high ϕ) increase incentives to liquidate. In terms of the Figure 3, increased disruption costs shift $E\Pi^L$ downwards.

The key of the theorem is that a higher scope for risk shifting (higher α) flattens the investment curve, as shown by the arrow to the thicker curve in Figure 3. This increases the lower bound of b, $\underline{b} = 1 - \alpha$ such that the probability is well defined. The amount that can be extracted by the remaining strategies is unchanged. Strategy (L), or early liquidation $(\sigma^\ell, 0)$, gives the bank $E\Pi^L = R\ell$. Partial zombie lending (or late liquidation) yields $E\Pi^{ZP}$. Both $E\Pi^{ZP}$ and $E\Pi^L$ are independent of b, and thus are flat. Thus, under a monopolistic behavior bank, increasing b decreases the profitability of lending for investment. Given that there is a profitable project, the bank would normally like to extend funds. However, if the bank decides to fund investment, the necessary repayment for this to be profitable would persuade the firm to increase the risk of the project past the point of profitability. Without funding investment, the bank can obtain all business-as-usual output via zombie-lending or partial zombie-lending. There is a conceptual difference between zombie-lending and liquidation. Even though the bank takes all the revenue when it decides to zombie-lend or partial zombie-lend, this is a very different strategy from liquidating. Under zombie-lending, the technology of the firm remains the same. When the firm is instead liquidated, disruption costs are paid, and a new entrepreneur can enact the (efficient) investment project.

The Case of a Financially Distressed Firm with Negative Operational Profits.

The previous sections analyzed the case of a financially distressed firm in debt overhang with positive operational revenues, i.e., with negative profits due to large debt repayments. In this section, we consider the case of a financially distressed firm with operational losses. Financial distress grants the bank monopolistic power. In the absence of this monopoly, the firm with operational losses would still be able to gather funds to enact efficient projects.

The model allows for operational losses by including a fixed cost in the firm's profits, κ. The firm's profits in this case are given by

$$\begin{array}{ll} \max_{\mathbb{1}_I} E\pi & = \max_{\mathbb{1}_I}\{y_0 - r - \kappa + \mathbb{1}_I(EV_i(b) - X) + (1-\mathbb{1}_I)p_0(y_0 - b - \kappa)\} \\ EV_i & = \max_i \left\{p_i(p_i^{-\alpha} - b) - \kappa\right\} \end{array} \tag{10}$$

Notice that the firm's profits can be negative in the first period, but the value of the firm can still be positive under the implementation of a profitable project. Moreover, even if the firm is having operational losses in period 1, the liquidation value may be positive if the investment project is carried out. As can be seen from Equation (10), the structure of the problems is mostly unaffected by these changes, but the set of efficient investment projects and the liquidation value need to be adapted. The following proposition characterizes these sets.

Proposition 2.8. *Let $(p_0, \epsilon, \phi, \alpha, \kappa) \in [0,1]^4 \times R_+$ and the firm's profits be given by Equation (10). If the firm is in debt overhang as in Proposition 2.2, when $\alpha \to 1$,*

1. $\ell \to \max\{0, \phi\left(\frac{\epsilon}{p_0} + 1 - 2\kappa\right)\}$.
2. *Liquidation is socially inefficient if* $2\kappa \leq \frac{\epsilon}{p_0} + 1$.
3. *The firm makes losses if* $\kappa \geq \frac{\epsilon}{p_0}$.
4. *Investment is efficient if* $b \in I'_X \equiv (0, \ell]$.

The liquidation value will be positive if the losses at $t = 1$ are compensated by profits at $t = 2$, after the company invests. An efficient investment project will be enacted as long as the investment cost is lower than the potential recovery value for the bank. Finally, if the firm has a profitable project, then it is inefficient to liquidate it, given that the firm is still viable and would benefit from restructuring. Liquidation is efficient when the net present value of the firm is negative, meaning that it will sustain operational losses even when enacting its best projects. Liquidation is inefficient when a firm has a project that will increase profits.

The following theorem characterizes the values of the profit functions for the bank when liquidation is socially inefficient. Strategy (I) is not an equilibrium strategy because the firm will increase the project's risk, rendering it non-profitable. Since (ZP) implies no change in the technology of the firm, it will yield negative profits for the bank in $t = 1$. Since the profits of liquidating the firm, (L), are bounded below by zero, this will be preferred. Finally, (Z) is dominated by (ZP) since it entails covering lower losses. Thus, there is no zombie-lending under operational losses. This is stated in Theorem 2.9.

Theorem 2.9. (No zombie lending with operational losses). *Given parameters* $(p_0, \epsilon, \phi, \alpha, \kappa, RX) \in [0,1]^4 \times R_+ \times I_X$*, when there is debt overhang as in Proposition 2.2 and the firm has operational losses (*$\kappa \geq \epsilon/p_0$*), then the bank chooses* (L)*, regardless of whether it is efficient or not.*

The Theorem states that a bank chooses to liquidate the firm if it has operational losses. This liquidation may be inefficient because the firm has a profitable project. However, the bank cannot induce it with a debt contract due to moral hazard, and thus strategy (I) is unfeasible. Therefore, the bank chooses (L), and even a viable firm is disrupted, paying social disruption costs ϕ.

2.5 The Case of a Financially Distressed Bank

According to empirical studies, zombie firms are more likely to be in lending relationships with undercapitalized banks (See Related Literature). Consider a simple modification to the game, shown in Figure 4. Bank capital is denoted by A and legal capital requirements by K. At the beginning of the game the bank starts with capital A_0. The bank can partly hide loans from firms that cannot pay back their contracted debt. Thus, the bank can disclose or not disclose this at $t = 0$. Liquidating the firm early entails disclosing the non-performing nature of the loan. At $t = 1$, the declared capital is subject to a shock, ψ, such that $A' = A - \psi$. When A' is lower than capital requirements, the bank goes bankrupt and obtains a value of ν. The shock ψ is distributed according to a smooth distribution with positive support, with accumulated density denoted by $G(\cdot)$. We assume that the bank will be capitalized at the end of the game if it decides to keep the firm afloat instead of liquidating at $t = 0$.[3]

If the bank does hide its loss, declared assets will be the original A_0 and will face the trade-off between strategies $(Z), (I)$, and (ZP). If the bank decides to liquidate early, its profits are

$$E\Pi^L = R\ell\left(1 - G\left(A_0' - K\right)\right) + G(A_0' - K)\nu. \tag{11}$$

By liquidating the firm, the bank writes down the value of the assets to $A_0' \leq A_0$. In this case, it instantly gets the liquidation value, ℓ. This will earn $R\ell$ in period 2. However, expected profits are now weighed by the probability of bank survival,

[3] This is a tractable way to include the preference for delaying liquidation in the bank. The modeling choice is discussed in Section 4.

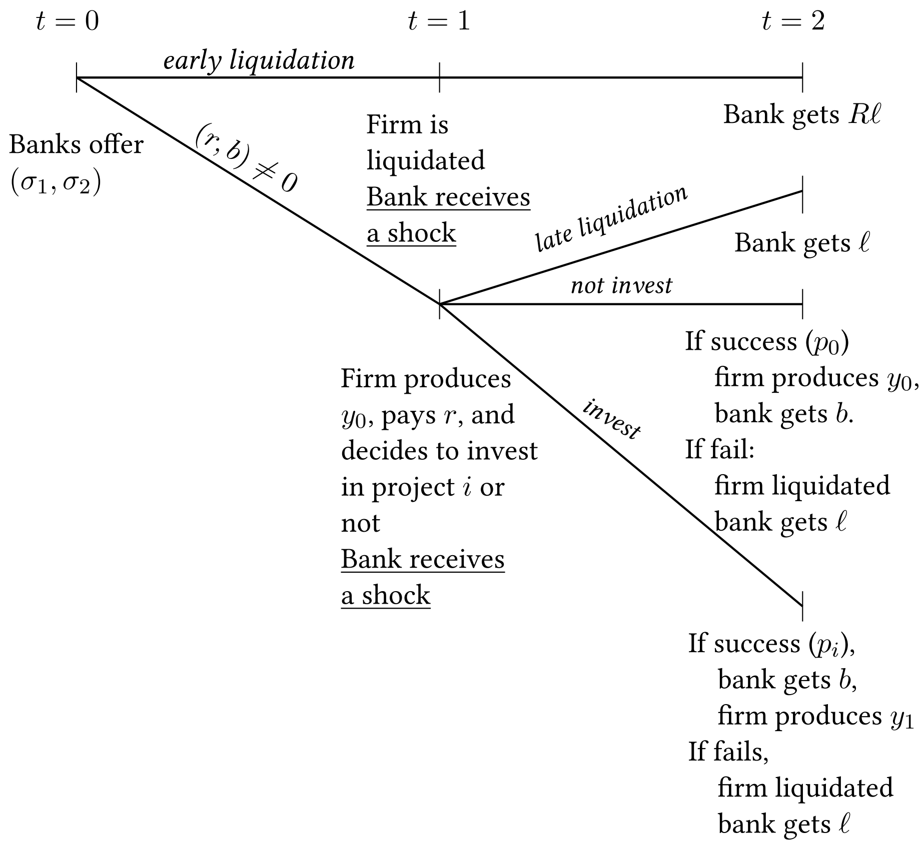

Figure 4: Timing of the model with a financially distressed bank.

$1 - G(A - \psi)$. In case of its own bankruptcy, the bank obtains a payoff equal to ν. Figure 5, shows the effect of acknowledging losses, which shifts the expected revenue function of liquidation downwards. Neither the funding investment of the zombie lending schedule is affected.

Financially Distressed Firm with Positive Operational Profits

The next theorem states that the conditions for the emergence of zombies are more relaxed when the bank is undercapitalized. This is because liquidation, which was an equilibrium strategy in the previous section, is no longer attractive for the bank if it entails a high risk of going bankrupt itself. The space of parameters for which (ZP) is an equilibrium is enlarged. In particular, the lower

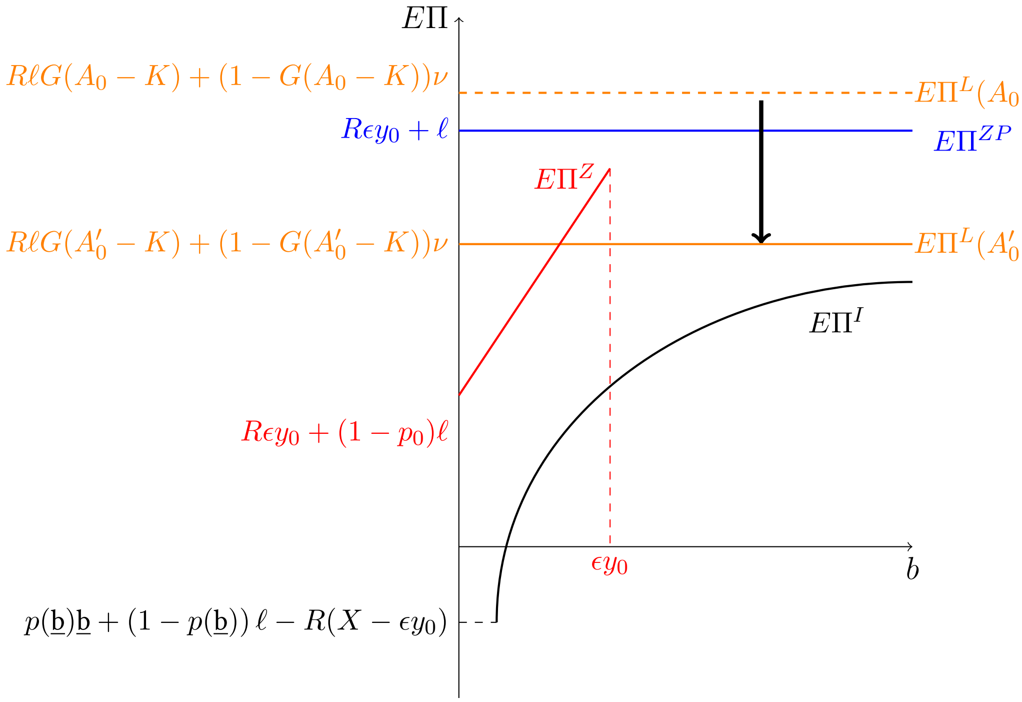

Figure 5: Profit functions of a financially distressed bank when $\alpha \to 1$. The arrow shows the effects of early liquidation.

the value of the bank when bankrupt, ν, the less likely it is that the bank will liquidate. In particular if $\nu \leq \phi$, the bank always chooses (ZP) over (L). High interest rates in this case make it more likely to have zombies via (ZP), given that it is more attractive to put the revenue in the risk-free technology, and there is no counterweight from the benefit of liquidating early, as the bank is at financial risk. In these cases, the bank keeps the firm alive for one or two periods by subsidizing its original debt payments but without funding any investment.

Theorem 2.10. (Zombie Firms with an undercapitalized bank and positive revenue). *Given parameters* $(p_0, \epsilon, \phi, \alpha, RX) \in [0,1]^4 \times I_X$*, and* $\nu \leq R\ell$*, under debt overhang as in Proposition 2.2, when* $\alpha \to 1$*, an undercapitalized bank's strategies* $(A \to K)$ *are*

- (Z) *if* $\epsilon \geq p_0 \left(\frac{\phi}{1-\phi}\right)$,
- (ZP) *if* $\epsilon \in [p_0 \left(\frac{\nu-\phi}{R+\phi}\right), p_0 \left(\frac{\phi}{1-\phi}\right)]$

- (L) *otherwise*

Financially Distressed Firm with Negative Operational Profits

When the firm has operational losses, the revenues for the bank are negative in the first period if it decides to keep the firm afloat. The bank revenues will continue to be negative if the efficient project is not enacted. Thus, the bank will sustain losses by following (Z) or (ZP). Notice that strategy (ZP) dominates strategy (Z), given that it allows the bank to liquidate in period 2 without facing any risk, thus limiting its losses in that period to zero. The bank's profits from liquidating are given by Equation (11). If $\nu \geq 0$, early liquidation will dominate the remaining strategies as the liquidation value of a firm is at least zero. Thus, the only possible way to observe partial zombie-lending is when the bank is financially distressed (i.e., low assets) and the value of its own bankruptcy, ν, is negative.[4]

Theorem 2.11. (Zombie Firms with an undercapitalized banks and negative revenue). *Given parameters* $(p_0, \epsilon, \phi, \alpha, \kappa, RX) \in [0,1]^4 \times R_+ \times I_X$*, under debt overhang as in Proposition 2.2, for an undercapitalized bank* ($A \to K$) *and a firm operating at a loss* ($\epsilon < \kappa p_0$)*, there exists a threshold* $\bar{\nu} < 0$ *when* $\alpha \to 1$ *such that*

- *if* $\nu \geq \bar{\nu}$ *the bank prefers* (L)*,*
- *if* $\nu \leq \bar{\nu}$*. the bank prefers* (ZP)*.*

The theorem states that for a sufficiently low value of bankruptcy of the bank, the bank covers the firm's losses in the $t = 1$, until it is able to liquidate the firm once it is capitalized. In other words, the bank trades a small loss from financing a firm's losses in order to reduce its expected bankruptcy risk. This is inefficient for two reasons. First, there exists a project that can increase social surplus that is not enacted. Secondly, the theorem shows that this is independent of any other parameters, so it is possible that liquidation is inefficient (e.g., in the case of a firm with positive net present value).

3 Policy Implications

We now discuss several policy implications that arise from the model.

[4]This can be interpreted as the stigma associated with bankruptcy, discussed in Section 4.

Debt Haircuts. Fundamentally, zombies arise because of the interaction between monopoly power due to debt overhang and moral hazard that limits the willingness of the bank to extend new funds. The bank is unwilling to grant further funds ($r < 0$) because the amount that it would then require in the next period to make it profitable (b) is so large as to not be compensated by the increase in risk. The bank then prefers to extract all the available funds from the firm. If capitalized, it declines the option to liquidate in order to avoid disruption costs.

In this context, partial debt forgiveness in the form of, for example, a debt haircut limits the capacity of the bank to extract funds from the firm and aligns the incentives for efficient risk-taking. Provided the haircut is large enough, the firm can regain access to the competitive market. Risk is reduced and expected social surplus is increased. This makes compensating the banks possible, thus increasing social welfare across the board. Notice that it is not optimal for the incumbent bank to privately offer any forgiveness, since it would lose its monopolistic gain and cannot ensure to profit from the new project, given that the firm may receive funds from the competitor bank. The next proposition states this result. The debt haircut relieves the firm from this locked-in relationship and the firm can then obtain funds on the competitive market to enact its efficient project.

In this way, society's surplus is increased: disruption costs are avoided and the efficient project is enacted.

Proposition 3.1. (Debt Haircuts.) *In the presence of zombies as in Theorem 2.7 or in inefficient liquidation in Theorem 2.9, there exists a debt haircut $\zeta \in [0, 1]$ on original debt D_0 and lump-sum transfers τ such that, at the new level of debt $D_0' = (1 - \zeta)D_0$, both the bank and the firm are better off. This haircut is not privately optimal for the bank.*

Notice that the proposition does not make any statement regarding an undercapitalized bank, since this policy could increase bankruptcy risk for such an agent; thus the effect on welfare is unclear.

Monetary Policy. Monetary policy usually works by lowering the opportunity cost of funds to induce investment. However, the monopolistic power of the bank creates a wedge between the opportunity cost of funds and the relevant rate for the firm. Moreover, interest rate changes affect the return on early liquidation. Lowering the interest rate makes it less profitable to liquidate. The next proposition states that there is a set of parameters such that reducing the interest rate has no effect on investment as the fundamental incentive problem of risk shifting under debt overhang is too strong.

Proposition 3.2. (Monetary Policy)

1. *Capitalized bank*
   (a) *Financially distressed firm (Theorem 2.7)*
      i. *If (Z) is the equilibrium, it remains so for any $R \geq 1$,*
      ii. *Lowering R increases the space of parameters for which (ZP) is an equilibrium, and never increases investment.*
   (b) *Financially and operationally distressed firm (Theorem 2.9)*
      i. *Inefficient liquidation remains an equilibrium for any R.*
2. *Undercapitalized bank*
   (a) *Financially distressed firm (Theorem 2.10).*
      i. *If (Z) is the equilibrium, it remains so for any R.*
      ii. *Increasing R decreases the space of parameters for which (ZP) is an equiliibrium, and this space decreases as ν decreases.*
   (b) *Financially and operationally distressed firm (Theorem 2.11)*
      i. *If (ZP) is the equilibrium, it remains so for any R.*

The first part of the Proposition states that if banks are capitalized and the firm has positive operational revenue, lowering the interest rate does not affect the incentives to lend for investment. This is because without competition, the required repayment is divorced from the policy rate. Moreover, lowering the interest rate decreases the incentives to liquidate early and enlarges the space of parameters for which partial zombie lending is an equilibrium. When the bank inefficiently liquidates a viable firm, lowering the interest rate does not shift the equilibrium strategy towards the socially superior strategy of funding investment. Finally, when the bank is too close to its own bankruptcy and thus chooses to perform partial zombie-lending to firms with operational losses, monetary policy is ineffective regardless of the interest rate. On the other hand, when an undercapitalized bank is paired with a financially distressed firm with positive revenue, the interest rate plays the expected role. Lower rates make survival lending less profitable, and thus liquidation is relatively more attractive. The effect decreases with the value of bank bankruptcy.

Bank recapitalization. When zombies arise with capitalized banks, recapitalizing the banks obviously has no effect. The only trade-off for banks exists between risk shifting and the obtainable revenues. Recapitalization could potentially have an effect under the conditions in Theorems 2.10 and 2.11. However, recapitalization may not be enough because it does not fix the trade-off between zombie-lending and funding investment given that the bank still has monopoly power and the firm can shift risk. Thus, at most, recapitalization will induce liquidation, which can be either efficient (when the net present value of the firm is negative) or inefficient. Inefficient liquidation only happens when the firm has a profitable project and positive operational revenues. The bank disrupts the firm before the efficient project is enacted, which entails a social loss. In other words, an operationally viable firm would benefit from renegotiation due to the sudden increase in debt, but this is not in the interest of the bank. This is stated in Proposition 3.3.

Proposition 3.3. (Bank Capitalization)

1. *When the bank is capitalized, the equilibrium strategy does not change, regardless on whether the firm has positive or negative operational revenue.*
2. *When the bank is not capitalized,*
   (a) *Small capitalizations can be ineffective,*
   (b) *If the firm is financially distressed (Theorem 2.10) and (Z) is the equilibrium, the strategy does not change. If the bank chooses (ZP), capitalization is ineffective if $\epsilon \geq \phi p_0 \left(\frac{R-1}{R-R\phi+\phi}\right)$.*
   (c) *When the firm has operational losses (Theorem 2.11), capitalization induces (L), efficient or inefficient*

Notice that in the condition of Theorem 3.3, in item 2b, the RHS increases as R increases. Thus, lower interest rates increase the range of parameters for which recapitalization is ineffective.

4 Discussion and Conclusion

Banks' lending to distressed firms at subsidized rates has been documented in several countries after the burst of bubbles. This paper explains why zombie-lending arises and continues to thrive even when renegotiation is possible, efficient projects are available, and banks are capitalized.

The model captures a fundamental conflict between lender and borrower in a double-decked incentive problem. The main drivers of this result are that the borrower becomes locked into a lending relationship with its incumbent bank and that it has access to a risk-shifting technology. Once the firm is in debt overhang, the incumbent bank can extract more funds from the firm than is socially optimal. Given the possibility of risk shifting, firms are not willing to use the fresh funds efficiently, and the bank anticipates this behavior. Previous explanations for delays in restructuring rely on distressed banks gambling for resurrection, strategic negotiation, or uncertainty. In the model presented, financially distressed firms are viable and have efficient projects, but these are not enacted when debt is large, even when the bank has funds. Moreover, the incentives of the actors do not allow the renegotiation process to solve the problem.

The mechanism present in the model suggests policy implications in stark contrast to previous papers and in line with several stylized facts. Partial debt forgiveness in the form of a debt haircut is shown to be privately suboptimal but necessary to restore investment. Fukuda and Nakamura [2011] shows that debt forgiveness was key in Japan. The model presented here explains why bank capitalization and monetary policy are not sufficient to restart investment as documented by Acharya et al. [2016] and Hoshi and Kashyap [2010]. First, strengthening banks is insufficient if insolvency regimes are hostile to the reorganization of indebted firms, as it only attacks one side of the incentive problem. This model generates a typology of firms that may receive subsidized lending and the conditions under which a bank may decide to pursue this strategy. In previous explanations of debt forbearance, firms are insolvent. Efficiency requires the liquidation of these firms, a process which may be delayed due to an insolvent bank gambling for resurrection. Here, however, firms are financially insolvent but still have positive operational revenue. The problem is that firms' debt became suddenly large, and the firms that could still grow failed to do so because of that debt. Optimality requires that these firms restructure their debt, but banks refuse to do so at any level of capitalization. Second, the literature that highlights the ineffectiveness of monetary policy relies on the zero lower bound and uncertainty that causes cash hoarding [Ito and Mishkin, 2006, Krugman et al., 1998]. We offer an alternative explanation. Lowering the policy rate decreases the opportunity cost for the bank, but not the effective rate facing the firm due to the debt overhang problem.

Additionally, there is only mixed evidence for the presence of risk shifting [5] and the literature has theoretically explored several mitigating factors.[6] The analysis of the lending mechanism in this paper suggests an explanation for the lack of empirical risk shifting: because the banks can foresee the potential misuse of their funds, they can choose not to lend in the first place.

There are some institutional solutions that facilitate the financing of distressed firms, such as debt-equity swaps, DIP financing [Kahl, 2002], or the bank gaining an equity stake in the firm in a way that allows it to control the risk. These

[5] See De Jong and Van Dijk [2007], Eisdorfer [2008], and Gilje [2016] for evidence.

[6] See Almeida et al. [2011] and Barnea et al. [1980] for evidence.

solutions can be understood within the model as requiring sizeable transaction costs or as entailing an acknowledgment of losses, and thus are addressed within the context of a financially distressed bank. Moreover, a bank may have limited managerial control over the firm it needs to take over.

Several assumptions are essential to these results. First, we assume that the shock is measure-zero. The choice of making the shock unexpected arises from the empirical fact that zombie firms tend to emerge after market bubbles burst. Alternatively, it can be argued that banks may also strategically restrain from taking risks even when a shock is expected [Nosal and Ordoñez, 2016]. Second, we assume that there are two banks. This is done merely for simplicity, since the second bank plays a role of an alternative to the incumbent in normal times. This assumption can be easily relaxed to a continuum of banks. Third, in the model debt contracts are non-contingent. Regardless, as long as the bank's monitoring technology is not perfect, the results hold to a certain degree. Fourth, undercapitalized banks are modeled as being transitorily fragile. The main reason for this choice is tractability. The fact that banks normally smooth out losses over many periods using provisions before liquidating the firm justifies this assumption.[7] Fifth, we assume that the firm has debt with only one bank, thus abstracting from potential conflicts of interest between lenders. Notwithstanding, syndicated loans have been growing increasingly more common, and a large number of creditors can behave collusively.[8] Sixth, we assume that the firm cannot enter bankruptcy voluntarily or, equivalently, that remaining a zombie is preferable to bankruptcy. In a way, this model assumes a stigma associated with bankruptcy.[9] Finally, for the case of undercapitalized banks, some degree of bank opacity is implicitly assumed; otherwise investors would price the asset loss in the market value of the bank.[10]

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[7] See for example Balboa et al. [2013] for evidence.

[8] See Gadanecz [2004] and Esty and Megginson [2003].

[9] Evidence of this can be found in Semadeni et al. [2008] and Gilson and Vetsuypens [1993].

[10] See BIS [2011], Flannery et al. [2004] and Huizinga and Laeven [2012] for evidence.

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5 Proofs

Proof of . Proposition 2.2. First we show that If D_0 is lower than $\bar{D}$, both banks offer $\hat{r} = -X$ and $\hat{b} = RX$. Assume it is not an equilibrium offer. If $\hat{r} \neq -X$, there is either not sufficient funds for investing or idle funds, so it is not an equilibrium. If the incumbent bank offers $\hat{b}' < \hat{b}$, then $\Pi < 0$, thus contradicting bank's incentive compatibility. If the incumbent bank offers $\hat{b}' > \hat{b}$, then the competitor can offer $\hat{b}'' = \hat{b}' - \varepsilon$, for arbitrarily small ε and have its offer accepted. Since there are enough funds to invest and the project is profitable, firm reacts using Equation (4) thus doing project $p(\hat{b})$. Lastly, if the incumbent bank offers to liquidate, $(\sigma^{\ell}, 0)$, then the firm gets zero profits. Then, the competitor can make an offer $(-X, \hat{b} - \varepsilon)$ such that $\Pi > 0$ and $\pi > 0$. The same logic applies to the competitor making offers. Thus, it is not an equilibrium.

If $D_0 > \bar{D} \equiv \gamma \{(y_0 - r - X(I) + \max_i\{EV_i(b), p_0 y_0\}\} \geq y_0$, where $\gamma > 1$. The claim is the bank will offer a strategy $(\sigma_1, \sigma_2) \in \{(\tilde{r}, \tilde{b}), (\tilde{r}', \sigma^{\ell}), (\sigma^{\ell}, 0)\}$ where $(\tilde{r}, \tilde{b}) \in \mathbb{R}$, chosen as to maximize profits as in Equation (1) subject to Equation (4), (6) and (8). Notice that if Equation (6) holds and the project is efficient (i.e., $EV_i \geq 0$), then equation (5) is slack. Now, suppose it is not an equilibrium. Then the incumbent bank can increase profits by offering $\tilde{b} + \mu$, which contradicts the maximization. The competitor bank can never make an offer below $\tilde{b}$, since Equation 7 never holds. Since $\pi(b) > 0$ for all b, then the firm is always strictly better accepting any offer from the incumbent bank. Similar argument holds for liquidation decision. □

Proof of . Proposition 2.3. By Bertrand-competition, both banks will offer $\hat{b} = RX$. By replacing in Equation 4, we obtain the desired result. $RX \geq 1 - \alpha$ ensures probabilities are well defined. □

Proof of . Proposition 2.4. The indirect profit function for the firm is given by

$$E\pi(b) = \left(\frac{1-\alpha}{b}\right)^{\frac{1-\alpha}{\alpha}} - \left(\frac{1-\alpha}{b}\right)^{1/\alpha} b$$

Outside value of not investing is $p_0 y_0$. By replacing $\hat{b} = RX$ and operating, we find that if $RX \in I_X$ investment is preferred. For probability to be well defined, we need that $RX \geq 1 - \alpha$. Maximizing social surplus is equivalent to maximizing welfare,

$$\max_b W(p(b)) = p^{1-\alpha} - b + (1-p)\ell \to \frac{\partial W}{\partial b} = \frac{\partial p}{\partial b}(b - \ell) - 1$$

Substituting by the functional form of $p'(b)$, it is immediate that if $b \leq \ell$ then derivative is positive. □

Proof of . Proposition 2.5. First I show that each of these strategies is an equilibrium. Then, I show that these are the only possible strategies.

1. First we show that (r^I, b^I) is an equilibrium strategy if the bank wants to fund investment. $r^I = \epsilon y_0 - X$ is an equilibrium given that any $r' \neq r^I$ is either not enough to fund investment or leaves idle resources. To find b^I, replace the reaction function of the firm in the problem of the bank and maximize,

$$\Pi = \left(\left(\frac{1-\alpha}{b}\right)^{1/\alpha} b + (1 - \left(\frac{1-\alpha}{b}\right)^{1/\alpha})\ell + Rr \right) = (1-\alpha)^{(1-\alpha)} b^{\frac{-1}{\alpha}} (b - \ell)$$

   The maximization yields,

$$-(1-\alpha)^{\frac{1}{\alpha}} \frac{b^{\frac{-1}{\alpha}-1}}{\alpha}(b-\ell) + (1-\alpha)^{\frac{1}{\alpha}} b^{\frac{-1}{\alpha}} = 0 \rightarrow b^M = \frac{\ell}{1-\alpha}$$

2. If the bank decides to zombie-lend, it sets $(r^Z = \epsilon y_0, b^Z = \epsilon y_0)$. Assume, on the contrary, that $r^Z > \epsilon y_0$, then the firm makes negative profits, thus violating feasibility. Assume $r^Z < \epsilon y_0$. Since bank's profits are increasing in r, $r' = r^Z + \varepsilon$ with arbitrary small ε, is feasible and yields higher profits.
3. Liquidating is trivially a strategy the bank finds it profitable to liquidate.
4. Setting $r^P = \epsilon y_0$ in the first period, there are no idle funds. Liquidation in the second period is trivially a strategy. Moreover, notice that (r^P, σ^ℓ) dominates (r^I, σ^ℓ).

Say the bank offers r'' such that $r^Z > r'' > r^I$, then there are either not enough funds to invest (in which case it cannot extract b^I) or idle funds. Thus, only possible equilibrium strategies are $\{(r^Z, b^Z), (r^I, b^I), (\sigma^\ell, 0), (r^P, \sigma^\ell)\}$. □

Proof of . Lemma 2.6. If the does not lend enough for investment, a project that increases welfare is not enacted. Liquidation is inefficient since implies disruption costs and missed efficient projects. From Proposition 2.3, all that remains to show now is that $\hat{p} = p(RX) \geq p^I(b^I)$. Since $RX \geq (1-\alpha)$, and if $(1-\alpha)^2 \geq \ell$, this is true. Under Proposition 2.4, the project at $\hat{p}$ is efficient and increases welfare. Thus, welfare is lower. □

Proof of . Theorem 2.7.
Relevant Sets. Let $\Omega = \Big\{(p_0, \alpha, \phi, \epsilon, RX) \in [0,1]^4 \times I_X, \Big\} \subset \mathbb{R}^5$, such that

$$\begin{aligned} y_0 &= p_0^{-\alpha} \in (0, \alpha]\,, \\ \ell &= \phi \left(\epsilon y_0 + \alpha (1-\alpha)^{\frac{1-\alpha}{\alpha}} (RX)^{-\frac{1-\alpha}{\alpha}} \right) \geq (1-\alpha)^2 \\ RX \in I_X &\equiv \left[(1-\alpha), \max \left\{ \ell, (1-\alpha) \left(\frac{\alpha p_0^\alpha}{\epsilon} \right)^{\frac{\alpha}{1-\alpha}} \right\} \right] \end{aligned}$$

Which ensures that probabilities are well defined and that investment is efficient. First, notice that

$$\ell = \phi\left(\epsilon y_0 + \alpha\left(1-\alpha\right)^{\frac{1-\alpha}{\alpha}}\left(RX\right)^{-\frac{1-\alpha}{\alpha}}\right) = \phi\left(\epsilon p_0^{-\alpha} + \alpha\left(\frac{1-\alpha}{RX}\right)^{\frac{1-\alpha}{\alpha}}\right)$$

Which, since $\lim_{\alpha\to 1}\alpha\left(\frac{1-\alpha}{RX}\right)^{\frac{1-\alpha}{\alpha}} = 1$, and $\lim_{\alpha\to 1} p_0^{-\alpha} = \frac{1}{p_0}$, converges to

$$\lim_{\alpha\to 1}\ell = \phi\left(1 + \frac{\epsilon}{p_0}\right) \tag{12}$$

Second, since $\lim_{\alpha\to 1}\left(1-\alpha\right)\left(\frac{\alpha p_0^{\alpha}}{\epsilon}\right)^{\frac{\alpha}{1-\alpha}} = +\infty$ and $\lim_{\alpha\to 1}\ell = \phi\left(1+\frac{\epsilon}{p_0}\right)$, then $\lim_{\alpha\to 1} I_X = \left[0, \phi\left(1+\frac{\epsilon}{p_0}\right)\right]$.

Part A. We now find conditions such that $E\Pi^Z \geq E\Pi^I$. Bank will prefer to offer $r^Z = \epsilon y_0$ and $b^Z = \epsilon y_0$ instead of $r^I = \epsilon y_0 - X$ and $b^I = \frac{\ell}{1-\alpha}$ if

$$R\epsilon y_0 + \epsilon p_0 y_0 + (1-p_0)\ell \geq R\epsilon y_0 - RX + p(b)b + (1-p(b))\ell$$

Replacing and simplifying,

$$RX + \epsilon p_0^{1-\alpha} - p_0\ell - \alpha\left(1-\alpha\right)^{\frac{2-\alpha}{\alpha}}\ell^{-\frac{1-\alpha}{\alpha}} \geq 0 \tag{13}$$

Taking limits in each term

$$\lim_{\alpha\to 1}\epsilon p_0^{1-\alpha} = \epsilon$$
$$\lim_{\alpha\to 1} p_0\ell = \phi(p_0+\epsilon)$$
$$\lim_{\alpha\to 1}\alpha\left(1-\alpha\right)^{\frac{2-\alpha}{\alpha}}\ell^{-\frac{1-\alpha}{\alpha}} = 0$$

Then, $E\Pi^Z \geq E\Pi^I$ if

$$RX + \epsilon - (p_0+\epsilon)\phi \geq 0$$

which holds for nonempty $\Omega' \in \Omega$.

Part B. Now we find conditions under which $E\Pi^Z \geq E\Pi^L$. This is true if

$$R\epsilon y_0 + \epsilon p_0 y_0 + (1-p_0)\ell \geq R\ell$$

Replacing and taking limits, when $\alpha \to 1$

$$\epsilon(R+p_0) \geq (R-1+p_0)\phi(p_0+\epsilon)$$

Remark #1: Notice that if $\epsilon \geq p_0 \frac{\phi}{1-\phi}$, the LHS grows faster than the RHS when changing R. Thus, if the condition holds for $R = 1$, will hold for any $R \geq 1$. Setting $R = 1$,

$$\epsilon(1 + p_0) \geq p_0^2\phi + \phi p_0 \epsilon \rightarrow p_0^2(-\phi) + p_0(1 - \phi)\epsilon + \epsilon \geq 0$$

The last equation is a quadratic equation in p_0, and is straightforward to show that is positive for any value of the parameters. Thus, $E\Pi^Z \geq E\Pi^L$ unconditionally.

Part C. Now we find conditions under which the bank prefers $E\Pi^Z$ to $E\Pi^{ZP}$. This is true if

$$E\Pi^{ZP} = \ell + Ry_0 \leq Ry_0 + p_0y_0 + (1 - p_0)\ell = E\Pi^Z$$

Simplifying and taking limits $\alpha \rightarrow 1$,

$$\epsilon \geq \frac{\phi}{1 - \phi} p_0 \tag{14}$$

Notice that this condition contains is exactly the same as in Remark #1.

Part D. From Part C, we know that if $\epsilon \leq \frac{\phi}{1-\phi} p_0$ then $E\Pi^{ZP} \geq E\Pi^Z$. Thus, we need to find conditions for $E\Pi^{ZP} \geq E\Pi^L$ and $E\Pi^{ZP} \geq E\Pi^I$. By taking limits, we find that $E\Pi^{ZP} \geq E\Pi^I$ when $\alpha \rightarrow 1$, unconditionally. Finally,

$$E\Pi^{ZP} = R\epsilon y_0 + \ell \geq R\epsilon y_0 + \epsilon p_0 y_0 + (1 - p_0)\ell = E\Pi^L$$

Reworking the conditions and taking limits,

$$\epsilon \geq \frac{R - 1}{R - \phi R + \phi} \phi p_0$$

which is lower than $\frac{p_0\phi}{1-\phi}$. Combining with the Part C, we establish that for $\epsilon \in [p_0 \frac{R-1}{R-\phi R+\phi}, p_0 \frac{\phi}{1-\phi}]$, $E\Pi^{ZP}$ is chosen in equilibrium.

□

Proof of . Proposition 2.8.

1. The liquidation value in this case is the profits in the first period plus the future stream of profits of enacting a project that is profitable,

$$\begin{aligned} \lim_{\alpha \rightarrow 1} \ell &= \lim_{\alpha \rightarrow 1} \max \left\{ 0, \phi \left(\epsilon y_0 + \alpha (1 - \alpha)^{\frac{1-\alpha}{\alpha}} (RX)^{-\frac{1-\alpha}{\alpha}} - 2\kappa \right) \right\} \\ &= \max \left\{ 0, \phi \left(\frac{\epsilon}{p_0} + 1 - 2\kappa \right) \right\} \end{aligned}$$

2. The net present value of the firm is the term in brackets, and thus if $\frac{\epsilon}{p_0} + 1 - 2\kappa \leq 0$ liquidation is efficient.

3. Profits in first period are $\frac{\epsilon}{p_0} - \kappa > 0 \rightarrow \kappa \geq \frac{\epsilon}{p_0}$.
4. The indirect profit function for the project is

$$E\pi(b) = \left(\frac{1-\alpha}{b}\right)^{\frac{1-\alpha}{\alpha}} - \left(\frac{1-\alpha}{b}\right)^{1/\alpha} b - \kappa$$

and the project will be efficient for the firm if $E\pi(b) \geq 0$. This is true if $b \geq (1-\alpha)\kappa^{\frac{-\alpha}{1-\alpha}} \rightarrow_{\alpha\rightarrow 1} 0$. For society, the project will be efficient if it increases social surplus under competitive markets,

$$\max_b W(p(b)) = p^{1-\alpha} - b + (1-p)\ell - \kappa$$

Taking the first order condition, project increases social welfare if $b \geq \ell$ since $p' > 0$ and $b \geq 0$. For the probabilities to be well defined the repayment $b \geq (1-\alpha)$. Thus, $b \in \left[\min\left\{1-\alpha, (1-\alpha)\kappa^{\frac{-\alpha}{1-\alpha}},\right\}, \ell\right] \rightarrow_{\alpha\rightarrow 1} (0, \ell]$.

□

Proof of . Theorem 2.9.

The profit functions for each strategy are

$$\lim_{\alpha\rightarrow 1} E\Pi^Z = \lim_{\alpha\rightarrow 1} \{R(\epsilon y_0 - p_0\kappa) + \epsilon p_0 y_0 + (1-p_0)\ell - \kappa\} \quad (15)$$

$$= R\left(\frac{\epsilon}{p_0} - \kappa\right) + \epsilon - \kappa + (1-p_0)\phi\left(\frac{\epsilon}{p_0} + 1 - 2\kappa\right) \quad (16)$$

$$\lim_{\alpha\rightarrow 1} E\Pi^I = \lim_{\alpha\rightarrow 1} \{R(\epsilon y_0 - \kappa) - RX + p(b)b + (1-p(b))\ell - \kappa\} \quad (17)$$

$$= R\left(\frac{\epsilon}{p_0} - \kappa\right) - RX - \kappa + \phi\left(\frac{\epsilon}{p_0} + 1 - 2\kappa\right) \quad (18)$$

$$\lim_{\alpha\rightarrow 1} E\Pi^{ZP} = \lim_{\alpha\rightarrow 1} \{R(\epsilon y_0 - \kappa) + \ell\} \quad (19)$$

$$= R\left(\frac{\epsilon}{p_0} - \kappa\right) + \phi\left(\frac{\epsilon}{p_0} + 1 - 2\kappa\right) \quad (20)$$

$$\lim_{\alpha\rightarrow 1} E\Pi^Z = \lim_{\alpha\rightarrow 1} R\ell = R\phi\left(\frac{\epsilon}{p_0} + 1 - 2\kappa\right) \quad (21)$$

Since the firm has operational losses, $\epsilon \leq \kappa p_0 \rightarrow \epsilon \leq \kappa$. Then $E\Pi^Z \leq E\Pi^L$, since it includes two negative numbers and since $p_0 \leq 1$, a term smaller than ℓ. $E\Pi^I \leq E\Pi^L$ since the extra terms in $E\Pi^I$ are negative.Thus, can only be partial zombie lending if (ZP) is greater than (L). Since $\epsilon \leq \kappa p_0$, and $R \geq 1$, $E\Pi^{ZP} \leq \ell \leq R\ell = E\Pi^L$. Notice that this is independent of the value of ℓ, since it can be positive (inefficient liquidation) or zero (efficient liquidation). □

Proof of. *Theorem 2.10.* Notice that profit functions for $E\Pi^Z, E\Pi^L, E\Pi^{ZP}$ given by Equations (16), (18) and (20) remain as in previous proof. $E\Pi^L$, instead of Equation (16) is now $R\ell(1-G(A-K))+G(A-K)\nu$. Say now the bank has very little capital, so $A \to K$, since is continuously non increasing in A, $G(\cdot) \to 1$. Thus, $E\Pi^L \to \nu = 0 \leq R\ell$. From Theorem 2.7, we know that bank prefers $E\Pi^Z$ to $E\Pi^L = R\ell$ and $E\Pi^I$ unconditionally. Moreover, $E\Pi^Z \geq E\Pi^{ZP}$ if $\epsilon \geq p_0\frac{\phi}{1-\phi}$. Finally, since $E\Pi^{ZP} \geq \nu \iff \epsilon \geq \frac{\nu-\phi}{R+\phi}p_0$. □

Proof of. *Theorem 2.11.* Notice that profit functions for $E\Pi^Z, E\Pi^L, E\Pi^{ZP}$ given by Equations (16), (18) and (20) remain as in previous proof. $E\Pi^L$, instead of Equation (16) is now $R\ell(1-G(A-K))+G(A-K)\nu$. Comparing Equations (21) and (20), it is clear (Z) is dominated by (ZP) since $\epsilon \leq \kappa p_0$. Moreover, from Equations (21) and (18), (ZP) dominates (I). When $A \to K$, $E\Pi^L \to \bar{\nu} < 0$. Thus, for $\bar{\nu} = E\Pi^{ZP}$, banks prefer to do (ZP) instead of (L), and viceversa if condition is not met. □

Proof of. *Proposition 3.1.* Welfare is maximized under competitive markets from Proposition 2.3. From Proposition 2.2, market structure depends on the level of debt. Suppose debt is $D_0 = \xi$. A haircut $\zeta > 0$ turns debt into $D_0' = (1-\zeta)\xi < \bar{D}(y_0)$. From Proposition 2.6, welfare is non decreasing. Increased surplus can be redistributed in a lump sum fashion using transferences τ such that both agents are better off. □

Proof of. *Proposition 3.2.* .

1. Capitalized bank
   (a) From Theorem 2.7
      i. Bank prefers to do (Z) if $\epsilon \geq p_0\frac{\phi}{1-\phi}$, at any level of R.
      ii. Bank prefers to do (ZP) if $\epsilon \in [p_0\phi\frac{R-1}{R-\phi R+\phi}, p_0\frac{\phi}{1-\phi}]$. Notice the lower bound of the set increases in R, and the upper bound is unaffected. Moreover, when $R \to 1$ the set becomes $[0, p_0\frac{\phi}{1-\phi}]$.
   (b) Conditions in Theorem 2.9), are unaffected by R.
2. Undercapitalized bank
   (a) From Theorem 2.10, (ZP) if $\epsilon \in [\frac{\nu-\phi}{R+\phi}p_0, p_0\frac{\phi}{1-\phi}]$. Increasing R increases the lower bound.
   (b) R does not affect any of the conditions for Theorem 2.11.

□

Proof of. *Proposition 3.3.* Under Theorem 2.7, and 2.9, the level of assets of banks is irrelevant, so it is trivially true. Under Theorem 2.10, the only strategy that is affected is (L). Given the continuity and monotonicity of G, there is a level of bank assets $\bar{\bar{A}} \geq A_0$ such that $R\ell G(\bar{\bar{A}} - K) \sim R\ell$. Thus, we revert to results from Theorem 2.7. Finally, from Theoreom 2.11, we know that $E^{ZP} \leq \ell$. Increasing A increases $E\Pi^L$ from $\nu 0$ to

ℓ. All other strategies remain dominated, and given continuity, (L) will be chosen. All that remaisn to show is that small capitalizations can be ineffective. Results follow from continuity and assuming any of the conditions for the chosen equilibrium strategies is slack.

□