Asymmetric Effects of Trade and FDI: The Role of Country Size and Bridge Multinational Production

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Abstract

Total gains from openness come through three channels: (i) trade, (ii) domestic multinational production (DMP), and (iii) bridge multinational production (BMP). We develop a quantitative theory to measure the effects of trade barriers and country size on the gains from openness through each of these channels. We show that these are sensitive to country size and larger in Europe than in South America. Country size is a crucial determinant of the contribution of each of these channels to the total gains from openness. DMP is more important in large countries, whereas the BMP channel is more important in small countries.

Keywords: Trade, Multinational Production, Bridge Multinational Production, South America, Europe.

JEL Codes: F12, F15, F23

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1 Introduction

International activity is concentrated on a few very large firms that produce in multiple countries. Multinationals and their foreign affiliates account for one-third of world output and GDP and two-thirds of international trade [Andrenelli et al., 2018].

This paper provides a quantitative assessment of the relationship between country size and gains from trade when including Multinational Production (MP), besides the standard gains from exporting. MP consists of two channels: Direct Multinational Production and Bridge Multinational Production. Direct Multinational Production (DMP) occurs when a multinational firm serves a foreign market by setting up a factory in that location. Bridge Multinational Production (BMP) occurs when a Multinational Firm sets up a plant in a third country and uses it as a bridge to export to a target destination.

The logic of the model is as follows. A firm's profits in a given country are a function of aggregate expenditure, which is determined, among other factors, by the market size. Given two countries with similar variable and fixed trade costs, a multinational plant will prefer to set up a plant in the larger country. As a result, a small country will attract less investment than a larger one. Suppose that a Japanese firm is interested in serving the Uruguayan market. The firm can export directly, but it may set up a plant directly in Uruguay (DMP) if transport costs are high. Alternatively, the firm may consider producing in Argentina, serving Uruguay via exports (BMP). In other words, the productivity required by a Japanese firm to start producing in the larger country is lower than that required to produce in Uruguay, given the same trade and efficiency costs. Size is a crucial variable in attracting foreign investment.

We focus on European and South American countries throughout the paper, as they present very different trade arrangements. While high trade barriers characterize South America, Europe has low trade barriers for the members of the EU. We first provide evidence for these regions. We show that small countries are relatively more open than large countries. Additionally, we find a differential effect between countries belonging to Mercosur (South American trade union) and the EU (European Union). Belonging to the EU increases trade openness, but membership to Mercosur does not. Furthermore, small countries in the EU benefit more than small countries in Mercosur.

Analyzing the differences in the gains from openness in these two regions for countries of different sizes provides information on the gains from integration. We find that FDI is negatively related to country size. Being part of the European

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Union increases the ability of countries to attract foreign firms, but being part of Mercosur may not. Small countries in Mercosur do not benefit from FDI as much as small countries in Europe.

We first show, in a stylized model, that the gains from trade when including BMP are larger for smaller countries. In order to assess the quantitative implications, we extend the Melitz [2003] model of trade with heterogeneous firms to a multi-country setting, where we include the possibility that firms engage not only in exporting but also in DMP and BMP. We quantitatively compare the performance of large and small countries in two regions: South America, where trade barriers are high, and Europe, where trade barriers are low.

We find that gains from openness in Europe are double those for South America (10.5% versus 5.3% of real GDP). The reason is that South America is relatively closed and thus benefits little from trade and MP. Moreover, we decompose the contribution of MP into DMP and BMP. DMP is more important for large countries, but BMP is more important for small countries. For example, in the Netherlands, MP explains 35% of the gains from openness, of which BMP explains almost two-thirds, while in Italy, MP explains 52% of the gains from openness, of which BMP explains only one-fifth.

We also investigate the gains that could be achieved in South America by improving the current degree of openness. When setting trade costs to the level of Europe, we find that all countries benefit from this reduction. The smallest country, Uruguay, benefits the most, increasing 30% manufacturing real GDP. Moreover, if, in addition, the variable costs of operating a multinational firm in South America were to decrease by 20%, gains for Uruguay would rise from 30% to 50%. These gains would be even more significant if the costs would only be lowered in Uruguay but not in the other South American countries. In this case, Uruguay would not have to compete with other countries to attract multinational firms. In the absence of BMP, any additional gains Uruguay would get by decreasing variable costs of operating a multinational are close to zero. BMP is crucial to attaining gains from higher efficiency.

Finally, the quantitative model shows that the differences between gains from trade and size vary highly between the two regions. In South America, the gains from trade are more homogeneously distributed (i.e., vary less with country size) than in Europe. The difference between the gains in real manufacturing GDP of a large and a small country is 8.5.pp in South America and 14.7pp in Europe. The greater heterogeneity in Europe stems from the higher degree of openness, allowing a small country in Europe to take more advantage of trade and MP than a small country in South America.

Related Literature. There is an extensive literature on the effects of trade and MP [Helpman, 1984, Horstmann and Markusen, 1992, Markusen, 1995, Markusen and Venables, 2000, Irarrazabal et al., 2013, Brainard, 1997, Carr et al., 2001, Markusen and Maskus, 2001, Yeaple, 2003]. This paper contributes to the role of BMP and country size to international trade gains.

Ekholm et al. [2007] developed a trade model with three countries to study the role of the export platforms. Our paper differentiates from that in that we allow for firm heterogeneity.

Ramondo and Rodríguez-Clare [2013] use a Ricardian trade model with multinational production to address the gains from openness. However, this model cannot address the effects of country size on the location of multinational firms since it assumes perfect competition and, as a result, does not model the fixed costs of MP.

Arkolakis et al. [2018] model trade and MP with monopolistic competition. They do not include fixed costs of setting up foreign firms. Fixed costs are essential to study the role that the size of a country plays in determining the location of multinationals. With fixed costs, there are increasing returns in production, which makes the size of a market a critical variable in making a location decision.

Tintelnot [2017] is closest to our paper. This paper includes a fixed cost of producing and performing MP and studies gains from openness (trade and MP) in a monopolistic competition set-up, but without focusing on the relationship between trade barriers and country size. We show the effect of BMP, country size, and trade barriers on output and trade across countries. In particular, our model allows comparing the benefits of trade, openness, and multinational production in South America versus Europe.

The paper is organized as follows: Section 2 presents empirical evidence on the relationship between trade, FDI, and country size. Section 3 and 4 present a simplified and the quantitative versions the model. Section 6 presents the results from the experiments, and Section 7 concludes. .

2   Motivating Facts.

This section presents empirical evidence on the relationship between trade, FDI, and country size for South America and Europe. The data for FDI flows is from World Bank Development Indicators, and FDI stock is obtained from UNCTAD statistics. The time frame for flows is 1990-2013, and for stocks is 1995-2003. We

perform the following regression,

$$
\begin{aligned}
y_{it} &= \beta_0 + \beta_1 POP_{it} + \beta_2 MER_i + \beta_3 MER_i \times POP_{it} + \\
&+ \beta_4 EU_i + \beta_5 EU_i \times POP_{it}. \quad\quad\quad\quad (1)
\end{aligned}
$$

where y_{it} is the outcome of interest (either $Trade/GDP$ or FDI/GDP), MER is a dummy variable that takes value 1 if the country belongs to the Mercosur Union (Argentina, Brazil, Paraguay, and Uruguay). POP is the natural logarithm of total population (when using from the WDI) or the labor force (in the case of UNCTAD data); EU is also a dummy variable which takes the value 1 if the country joined the European Union before 2000. Finally, we include the interaction of the two regional dummies and the population variable and year fixed effects.

Panel A of Table 1 presents the results of running regression 1 on two different samples using $Trade/GDP$ as the outcome variable. Small countries benefit the most from trade and are relatively more open than large countries. As expected, β_1 has a negative sign and is significant. The coefficients for Mercosur and EU are both positive, implying that countries in these regions have a higher $Trade/GDP$ than the rest of the world. However, the coefficient of Mercosur is not statistically significant. The coefficients of the interaction terms have the expected negative signs, which means that small countries in these regions have larger $Trade/GDP$. However, the coefficients for Mercosur are not statistically significant. In summary, $Trade/GDP$ has a negative relation with country size. Countries belonging to the European Union are positively associated with higher $Trade/GDP$, but belonging to Mercosur does not have a significant effect. Finally, the estimated coefficient of the interaction between region and country size (β_3 and β_5) suggests that small countries in the EU are more open and can benefit more from trade than small countries in Mercosur.

Panel B of Table 1 presents the results of running equation 1 using as the outcome variable net FDI inflows as a percentage of GDP (columns 1 and 2), or using as outcome variable the stock of FDI as a percentage of GDP (columns 3 and 4). The signs of the parameters of interest of equation 1 are in line with those of trade. The effect is negative and statistically significant for population. The coefficient for Mercosur is negative and slightly significant (for FDI stock/GDP) or not significant (for net FDI inflows/GDP). For the EU, the results are positive and significant. These results suggest that while being part of the European Union may increase the ability of countries to attract foreign firms, being part of Mercosur may not. The estimated coefficients of the interaction term yield consistent results: negative and significant for EU, and not significant for Mercosur indicating that small countries in Mercosur may not benefit from FDI as much as those in Europe.

Panel A	Trade/GDP	
	Full Sample	Excluding small countries
	(1)	(2)
Population	-4.15***	-5.78***
	(0.33)	(0.39)
Mercosur	21.35	-4.98
	(49.02)	(49.10)
Mercosur× Population	-3.013	-1.39
	(2.92)	(2.92)
European Union	114.59***	87.72***
	(32.99)	(33.16)
European Union× Population	-6.88***	-5.23***
	(2.00)	(2.01)
Adj. R-squared	0.06	0.07
N	4519	4246

Panel B	FDI/GDP flow		FDI/GDP stock	
	Full Sample	Excluding small countries	Full Sample	Excluding small countries
	(1)	(2)	(3)	(4)
Population	-0.81***	-0.74***	-5.65***	-6.36***
	(0.09)	(0.09)	(0.63)	(0.73)
Mercosur	-14.01	-12.93	-77.11*	-83.39*
	(13.38)	(11.07)	(44.02)	(44.42)
Mercosur× Population	0.77	0.70	7.19	7.90
	(0.79)	(0.66)	(4.77)	(4.82)
European Union	72.98***	74.15***	138.01***	131.74***
	(9.37)	(7.776)	(26.493)	(26.84)
European Union× Population	-4.25***	-4.32***	-13.74***	-13.04***
	(0.57)	(0.47)	(3.01)	(3.04)
Adj. R-squared	0.05	0.07	0.04	0.04
N	4283	4061	3430	3267

Table 1: Panel A. The dependent variable is Trade/GDP. The sample includes all countries in the WDI sample. Column (2) excludes countries with less than 100 thousand inhabitants. Panel B. The dependent variable is Net FDI inflow/GDP for the first two columns and FDI stock/GDP for the last two columns. The sample includes all countries in the WDI and UNCTAD sample. All regressions include year fixed effects. $***p < 0.01, **p < 0.05, *p < 0.1$.

3   Simple model

This section presents a simple version of the model to illustrate the relationship between BMP and country size. We extend Helpman et al. [2004] to allow for BMP, and show that small countries benefit more from openness in this context.

There are N countries and $H + 1$ goods. Goods are produced using only labor. One of the goods is homogeneous, whereas the remaining H are differentiated. Each country i is endowed with L_i units of labor.

The homogenous good is produced with one unit of labor. To produce differentiated goods in country i, a firm needs to pay a fixed cost of entry f_E, measured in labor units. Each potential entrant draws a draw a, of labor per unit output, from a distribution $G(a)$. Thus, $1/a$ represents the productivity level. Depending on the draw, the firm will decide whether to produce or to stay out of the market. If it chooses to produce, it will bear fixed labor costs f_D. If the firm also wishes to serve an international market, it can export or set up multinational production.

If the firm wishes to export, it will have to pay, on top of f_D, a fixed cost f_X per foreign market served, plus iceberg costs $\tau^{ij} > 1$ in order to take 1 unit from country i to country j. If, on the other hand, it wishes to serve the market via MP, it will need to pay f_M per market. There is thus a proximity-concentration tradeoff: either the firm can set up a plant and produce in the foreign market (losing concentration) or pay transportation costs while exporting.

The representative agent uses a proportion β of her income to consume the homogenous good, which we set as the numeraire. She has CES utility function with parameter $\epsilon = \frac{1}{1-\alpha} > 1$ that generates a demand for variety i given by

$$q_i = \frac{\beta E_i}{\int_0^{n_i} p^i(\nu)^{1-\epsilon} d\nu} \equiv A^i p^{-\epsilon}$$

A monopolistically competitive producer will sell at the price $p = \frac{a}{\alpha}$. For a consumer, the price will either be that, or $\tau^{ij} \frac{a}{\alpha}$ for imported goods from country j.

For simplicity of exposition, we assume that the origin of ownership, location of plant and place of consumption determine a variety. Thus, a firm can do BMP, FDI, and export to the same market since exported goods, goods produced via MP, and goods produced by BMP, are considered different varieties for the consumer.

Defining $B = \frac{(1-\alpha)}{\alpha^{1-\epsilon}} A^i$, the zero profit conditions for each individual activity are

$$\pi_D^{iii} = a^{1-\epsilon}B^i - f_D^i \qquad\qquad \to a^D = \left(\frac{f_D^i}{B^i}\right)^{\frac{1}{1-\epsilon}}$$

$$\pi_X^{jii} = (\tau^{ij}a)^{1-\epsilon}B^j - f_X^j - f_D^i \qquad \to a^X = \left(\frac{f_X^j + f_D^i}{\tau^{ij\,1-\epsilon}B^j}\right)^{\frac{1}{1-\epsilon}}$$

$$\pi_M^{kki} = a^{1-\epsilon}B^k - f_M^k \qquad\qquad \to a^M = \left(\frac{f_M^k}{B^k}\right)^{\frac{1}{1-\epsilon}}$$

$$\pi_B^{jki} = (\gamma^j\tau^{jk}a)^{1-\epsilon}B^j - f_M^k - f_X^j \qquad \to a^B = \left(\frac{f_M^k + f_X^j}{(\gamma^j\tau^{jk})^{1-\epsilon}B^j}\right)^{\frac{1}{1-\epsilon}}$$

Where π^{jki} is a company of country i, with a manufacturing plant in country k selling in country j.

We assume make two assumptions. First, we assume that $f_M > f_D$, implying that setting a multinational firm requires a higher cost than setting a company in the domestic market. Alternatively, can be thought that in order to create a multinational subsidiary the company needs to have a domestic plant, and thus $f_M - f_D$ is the incremental cost of setting a firm elsewhere. Second, we assume that $\tau^{jk}\gamma^k > 1$, which ensures that the productivity gains from setting a multinational elsewhere do not offset trade costs completely.

Under these assumptions, $a^{activity}$ with $activity = \{D, X, M, B\}$ are the cut-off productivities to perform domestic production, exporting, multinational production and bridge multinational production. All companies with $a < a^T$ will perform each activity. The previous assumption on fixed costs ensures that the companies always perform domestic production first, but it is possible for a firm to export and not do M or to have a multinational plant but not to export.

Expected profits for an entrant must be zero in equilibrium,

$$B^i V(a^D) + \sum_{i\neq j}\left\{(\tau^{ij})^{1-\epsilon}V(a_{ij}^X)B^j + V(a_{ij}^M)B^j + \sum_{k\neq i}(\gamma^k\tau^{jk})^{1-\epsilon}V(a_{ij}^M)B^k\right\} = EC$$

Where we define, for simplicity of notation, $V(a) = \int_0^a y^{1-\epsilon}dG(y)$, and expected costs, EC, are given by

$$EC = f_E + \int_0^{a^D} f_D^i dG(a) + \sum_{j\neq i}\left\{\int_0^{a^X}(f_X^j + f_D^i)dG(a) + \int_0^{a^M} f_M^j dG(a) + \sum_{k\neq i}\int_0^{a^B} f_M^k + f_X^j dG(a)\right\}$$

.

Symmetric Case. Now we turn to the almost symmetric case, where countries differ in size but all remaining parameters are the same across countries: the productivity distribution G, transport costs $\tau^{ij} = \tau \quad \forall i \neq j$, and fixed costs. Under these circumstancies, the cutoff productivity is the same for all countries and $B^i = B$ for every i. Thus, cutoffs are now

$$a^D = \left(\frac{f_D}{B}\right)^{\frac{1}{1-\epsilon}} \tag{2}$$

$$a^X = \left(\frac{f_X + f_D}{\tau^{1-\epsilon}B}\right)^{\frac{1}{1-\epsilon}} \tag{3}$$

$$a^M = \left(\frac{f_M}{B}\right)^{\frac{1}{1-\epsilon}} \tag{4}$$

$$a^B = \left(\frac{f_M + f_X}{(\gamma\tau)^{1-\epsilon}B}\right)^{\frac{1}{1-\epsilon}} \tag{5}$$

with

$$EC = BV(a^D) + (N-1)\tau^{1-\epsilon}V(a^X)B + (N-1)V(a^M)B + (N-1)^2\frac{N}{2}(\gamma\tau)^{1-\epsilon}V(a^M)B \tag{6}$$

We can see the profits function in Figure 1. In the x-axis is $a^{1-\epsilon}$, which decreases in a. Thus, higher values imply higher productivity. We assume that varieties are determined by the countries of ownership, production, and consumption; thus, all firms with $a^{1-\epsilon} < a_D^{1-\epsilon}$ do not produce, firms $a_D^{1-\epsilon} < a_X^{1-\epsilon}$ produce domestically but not export and so on. The profits from exporting and BMP are less steep due to trade costs, but different between each due to the impact on productivity of multinational production. Profits of domestic production and MP are parallel since the countries are symmetric.

The next proposition characterizes the number of entrants in each country.

Proposition 1. *The the number of entrants is*

$$n_E^i = \frac{(1-\alpha)\beta}{A}\{((N-1)v_{MX} + v_D)L_i - v_{MX}\sum_j L_j\}$$

for $\frac{L_i}{\sum_j L_j} > \frac{v_{MX}}{(N-1)v_{MX}+v_D}$, where A is a constant and $v_D = V(a_D)$ and $v_{MX} = \tau^{1-\epsilon}V(a_M) + V(a_X) + \tau^{1-\epsilon}\gamma V(a_B)N(N-1)/2$ Thus,
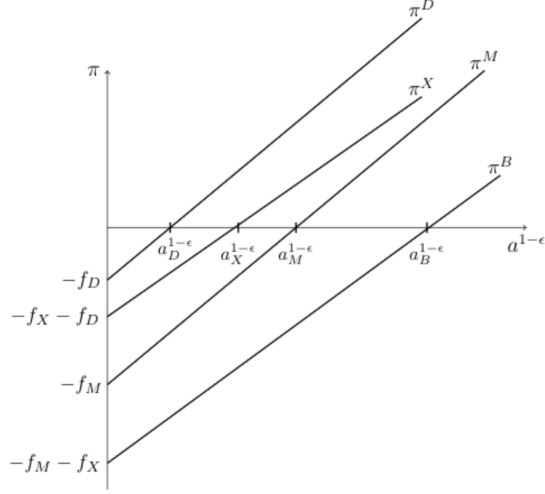
Figure 1: The cutoffs a_D, a_X, a_M, a_B represent the productivities at which the firm does domestic production, exports, multinational and bridge multinational. Firms below $a_D^{1-\epsilon}$ do not produce, and firms above $a_B^{1-\epsilon}$ do all activities.

- *Small countries benefit proportionally more from B,* $\frac{\partial n_E^i / L^i}{\partial v_{MX} \partial L_i} \geq 0$

- *Effect of trade costs is worse for small countries proportionaly,* $\frac{\partial n_E^i / L_i}{\partial \tau \partial L_i} < 0$

- *Productivity gains are proportionaly better for small countries,* $\frac{\partial n_E^i / L_i}{\partial \gamma \partial L_i} > 0$

Proof. See Appendix A.1. ◻

The proposition requires the countries to be not that different from each other in terms of size, for ensure that there is positive entry in each country. The intuition for the results is as follows. Notice first that Larger markets have a large number of firms, as standard in the literature. Second, small countries benefit more from BMP than large countries because BMP allows small countries to attract firms that will serve third-party markets. Thus, the potential market increases for multinationals willing to locate in a small country. For the same rationale, higher trade costs are exceedingly hurtful for small countries via the BMP channel, and multinational productivity gains are advantageous.

4 Quantitative Model

The model extends Melitz [2003] by including the possibility of direct multinational production (MP) and bridge multinational production (BMP).

The world economy consists of $i = 1, ..., N$ countries with population L_i. There is a representative consumer and a large mass of potential firms in each of them.

All goods in the economy are tradable, and there are two sectors of production. Sector 0 produces the homogenous good, and Sector 1 produces the differentiated good. The homogeneous good acts as the numeraire of the economy with price P^0.

Differentiated goods come from a continuum, indexed by $\omega \in \Omega$. Each differentiated good is produced by a firm with a given productivity that transforms the sole factor of production, labor, into the good. Three dimensions describe differentiated goods: the place of consumption, the site of production and the location of the owner of the firm. For example, $q_{ijk}(\omega)$ is the quantity of the good with variety ω consumed in country i, produced in country j by a firm from country k.

4.0.1 Consumers

In each country there is a representative consumer that supplies labor inelastically and who has Cobb-Douglas preferences:

$$U_i = q_{i,0}^{\delta_0} q_{i,1}^{(1-\delta_0)} , \tag{7}$$

where δ_0 is the share of the homogeneous good in total consumption and $q_{i,1}$ is a Dixit-Stiglitz aggregator:

$$q_{i,1} = \left(\int q_i(\omega)^{\frac{(\sigma-1)}{\sigma}} d\omega \right)^{\frac{\sigma}{\sigma-1}} ,$$

where $\sigma = \frac{1}{1-\rho} > 1$ is the elasticity of substitution between varieties and $q_{i,1}$ are all the varieties consumed in country i.

The demand functions for each sector are

$$q_{i0} = \frac{\delta_0 E_i}{P_i^0}$$
$$q_{i1} = \frac{(1-\delta_0) E_i}{P_i^0} \tag{8}$$

where P_i^0 is the aggregate price index in country i including the homogeneous good sector and E_i is the aggregate expenditure in country i. Define the expenditure in the differentiated good sector as $(1 - \delta_0)E = E^1$, where E is total expenditure. Then, the demand for each variety ω is given by:

$$q_{ijk}(\omega) = \frac{E_i^1}{P_i} \left(\frac{p_{ijk}(\omega)}{P_i} \right)^{-\sigma} , \tag{9}$$

where E_j^1 is the aggregate expenditure of country i in differentiated goods and $P_i = [\int_{\omega \in \Omega} p_{ijk}(\omega)^{1-\sigma} d\omega]^{1/(1-\sigma)}$ is the aggregate price in the differentiated good sector in country i. Note that in this case Ω is the set of goods consumed in the country, including those produced by domestic firms, those produced by foreign firms operating in the country, and those imported. The demand of good $q_{ijk}(\omega)$ is increasing in total expenditure and the aggregate price of the country where the good is consumed (E_i^1 and P_i), and decreasing in the price of the good (p_{ijk}).

4.1 Homogeneous good

Each country has an exogenous endowment z_i of the homogeneous good. This good is traded without any cost, thus equalizing its price across countries. We will denote the price of the homogeneous good as P_0. Each country will be an exporter or importer of this good depending on whether the domestic supply of the good is bigger or smaller than domestic demand.

The reason for introducing the homogenous good sector is that it allows our model to have countries with trade deficits in the differentiated good while also maintaining capital account deficits. Otherwise, trade deficits would require capital account surpluses and vice versa, which would affect firms' profits.

4.2 Differentiated good sector

Throughout the paper, we make two assumptions:

Assumption 1. *A variety is defined by the country of origin of the firm and the country where the good is produced.*

Assumption 2 *Any firm from country i performing an activity has to pay the domestic cost of producing in i.*

Assumption 1 allows treating a good produced by an Uruguayan owner, by a firm in Uruguay and exported to Brazil, and a good produced by the same Uruguayan owner but produced in Brazil for the domestic market as different varieties. The

fact that the production location also determines varieties simplifies the solution of the model by allowing to treat each activities' profits independently. In other words, the results from Melitz [2003] are extended by including MP and BMP as additional activities.

Without Assumption 1, competition between countries for attracting MP is increased, and thus BMP becomes an even more important factor to attract multinational firms. Thus, results from the quantitative section are a lower bound. In the results section we discuss the role of Assumption 1. Assumption 2 ensures that no firms will export or do MP and not sell in the domestic country.

4.2.1 Firms

Firms can engage in four activities: Domestic Production, MP, BMP, or Exporting. Firms maximize their variable profits of a firm from country i for a given activity,

$$\max_{p(\omega)} \pi = p(\omega)q(\omega) - c(\omega) , \tag{10}$$

where $q(\omega)$ was defined in Equation (9), and the price is given by a markup over variable costs.

To produce the differentiated good the only input used is labor. Firms pay a fixed entry cost κ_i^e to observe a productivity draw ϕ from a Pareto distribution. The cost is denominated in labor units; thus the effective cost is $w_i \kappa_i^e$, where w_i is the wage in country i. After observing the draw, firms decide whether to produce or not. If a firm chooses to produce, it can engage in four activities

1. Selling domestically. In this case, firms need to pay a fixed cost of operation κ_i^D, also denominated in labor units. In addition, firms also need to pay the variable cost of production. The variable cost of selling domestically $q_{iii}(\omega)$ units of the good is:

$$c_{iii}(\omega) = \frac{w_i}{\phi} q_{iii}(\omega) .$$

Thus, a firm will sell domestically if

$$\pi_{iii}(\phi) = \frac{E_i^1 P_i^{\sigma-1}}{\sigma} \left(\frac{\rho\phi}{w_i} \right)^{\sigma-1} - \kappa_i^D w_i \geq 0 \tag{11}$$

We denote the productivity cutoff associated with zero profits in the domestic market as ϕ_{iii}^{*0}.

2. Exporting from the domestic country. To export, firms need to pay a fixed cost independent of the selling destination and an iceberg type cost that is partner specific. Firms producing in country i and exporting to country j pay a fixed cost of exporting, $w_i \kappa_i^X$, and an iceberg cost per unit sold, $\tau_{ij} \geq 1$. In other words, in order to send one unit of the good from country i to country j, the firm needs to send τ_{ij} units. In this case, the variable cost of exporting $q_{jii}(\omega)$ units to country j is

$$c_{jii}(\omega) = \frac{\tau_{ij} w_i}{\phi} q_{jii}(\omega) .$$

The profits from a firm from country i exporting to country j is given by

$$\pi_{jii}(\phi) = \frac{E_j^1 P_j^{\sigma-1}}{\sigma} \left(\frac{\rho \phi}{w_i \tau_{ij}} \right)^{\sigma-1} - \kappa_i^X w_i \tag{12}$$

The cutoff for the exporting activity will be denoted ϕ_{jii}^{*0}.

3. Direct Multinational Production (DMP). In this case, the firm sets up a plant in a foreign country and engages to satisfy the same domestic foreign market. In this case, the firm obtains productivity gains by a factor of γ_{ij} that is partner specific. The new productivity for a firm from country i producing in country k is $\frac{\phi}{\gamma_{ki}}$. In addition, a firm from country i producing in country k has to pay a fixed cost $w_k \kappa_k^{MP}$ which is independent from the source country. Note, however, that the wage is that of the country where the firm is producing.

We assume that $\kappa_k^{MP} \geq \kappa_k^D$, implying that the cost includes setting up the plant, logistics, and extra costs to start it in a different country.

The variable cost of producing in country k to serve that very own market k, for a firm from country i, q_{kii} is given by

$$c_{kki}(\omega) = \frac{\gamma_{ki} w_k}{\phi} q_{kki}(\omega) .$$

The profit for a firm from country i performing DMP in country k is given by

$$\pi_{kki}(\phi) = \frac{E_k^1 P_k^{\sigma-1}}{\sigma} \left(\frac{\rho \phi}{w_k \gamma_{ki}} \right)^{\sigma-1} - \kappa_k^{MP} w_k \tag{13}$$

The zero profit productivity cutoff is denoted as $\phi_{kki}^{*,0}$ for DMP.

4. Bridge Multinational Production (BMP). Finally, a firm from country i that is produces in country k has the option to export to a third country, j. In this case,

the firm will have to pay an extra fixed cost of exporting $w_k \kappa_k^X$ plus the iceberg costs τ_{jk}. The variable cost of producing and exporting to a third country per $q_{jki}(\omega)$ units of the good is

$$c_{jki}(\omega) = \frac{\tau_{jk}\gamma_{ki}w_k}{\phi}q_{jki}(\omega) \, .$$

The profit for a firm from country i, producing in country k and selling in country j is given by:

$$\pi_{jki}(\phi) = \frac{E_j^1 P_j^{\sigma-1}}{\sigma}\left(\frac{\rho\phi}{w_k\gamma_{ki}\tau_{jk}}\right)^{\sigma-1} - \kappa_k^x w_k \tag{14}$$

We denote the productivity level that makes the above equation equal to zero as ϕ_{jki}^{0*}.

4.3 Cutoffs

In the previous Section we showed the productivity cutoffs each of the activities indendependently: ϕ_{iii}^{0*} (domestic production), ϕ_{kji}^{0*} (bridge multinational produciton), $\phi_{kki}^{*,0}$ (multinational production), and ϕ_{jii}^{0*} (exporting). We will call the cutoffs "well defined" if the cutoff of domestic activities is lower than the cutoff of exporting and the cutoff of multinational production, and the cutoff of bridge production is higher than the one of direct multinationals production. In this case, the effective cutoffs are equal to the independent cutoffs, $\phi_{iii}^* = \phi_{iii}^{0*}, \phi_{kki}^* = \phi_{kki}^{0*}, \phi_{kji}^* = \phi_{kji}^{0*}$.

However, we also allow for the possibility that one firm entails losses in one activity but compensates with profits from sales in another. For example, the firm may sustain losses in the domestic market but compensate with exporting. In this case, $\phi_{iii}^{0*} > \phi_{kki}^{0*}$. In this case the effective cutoff will be $\phi_{kki}^* = \phi_{iii}^*$, where the cutoff is the solution to

$$\pi_{iii}(\phi_{iii}^*) + \sum_{k\in K^X} \pi_{kii}(\phi_{iii}^*) = 0 \tag{15}$$

where K^X is the set of countries for which the domestic cutoff is higher than the exporting cutoff. The remaining effective cutoffs, for MP (ϕ_{kki}^*) and BMP (ϕ_{kji}^*) need to ensure that the firm can sustain losses in the domestic market that are compensated by profits from exports, multinatonal production, or bridge multinational production. The algorithm in Appendix A.5 ensures that all cutoffs are well calculated to maximize profits.

The profit made by a firm from country i is given by:

$$\pi_i(\phi) = \pi_{iii}(\phi) + \sum_{k \neq i} \pi_{kii}(\phi) I_{kii}^X + \sum_{k \neq i} \pi_{kki}(\phi) I_{kki}^M + \sum_{k \neq i} \sum_{j \neq, ik} \pi_{jki}(\phi) I_{jki}^B, \quad (16)$$

where I_{kii}^X is an indicator function that takes the value 1 if $\phi > \phi_{kii}^*$ and 0 otherwise, I_{kki}^M is an indicator function that takes the value 1 if $\phi > \phi_{kki}^*$ and 0 otherwise, and finally I_{jki}^B is an indicator function that takes the value 1 if $\phi > \phi_{jki}^*$ and 0 otherwise.[1] It is straightforward that since profits from every activity increase in ϕ, more productive firms make higher profits. Therefore if the productivity is high enough, a firm performs all the activities.

4.4   Productivity distribution

Productivities are drawn from a Pareto distribution density function given by $g_i(\phi) = \alpha_i \frac{(\phi_i^m)^{\alpha_i}}{\phi^{\alpha_i+1}}$ where ϕ_i^m is the scale parameter and α_i is the shape parameter . Since only firms with productivities above ϕ_{iii}^* will produce in country i, then the equilibrium distribution of productivities of domestic firms is

$$\mu_i(\phi) = \frac{g_i(\phi)}{1 - G(\phi_{iii}^*)} \text{ if } \phi \geq \phi_{iii}^*, \quad (17)$$

and zero otherwise. The probability of producing is given by $\theta_{iii} = 1 - G(\phi_{iii}^*)$ and the probability of performing other activities conditional on producing is:

$$\text{Exporting to country } k \Rightarrow \theta_{kii} = \frac{1 - G(\phi_{kii}^*)}{1 - G(\phi_{iii}^*)}$$

$$\text{MP in country } k \Rightarrow \theta_{kki} = \frac{1 - G(\phi_{kki}^*)}{1 - G(\phi_{iii}^*)}$$

$$\text{BMP in } k \text{ to sell in } j \Rightarrow \theta_{jki} = \frac{1 - G(\phi_{jki}^*)}{1 - G(\phi_{iii}^*)}$$

The average productivity for each activity is:

$$\tilde{\phi}_{jki} = \left[\int_{\phi_{jki}^*}^{\infty} \phi^{\sigma-1} \mu_i(\phi) d\phi \right]^{1/(\sigma-1)} \quad (18)$$

---

[1]There are no exporting or MP cutoffs in the calibrated model economies lower than the domestic cutoff. However, there are some BMP cutoffs smaller than the MP cutoffs that require the recalculation of the cutoffs.

for all i, j and k. Notice that $\tilde{\phi}_{jki}$ only depends on the cutoff productivity.

As in Melitz [2003], we consider that, for each activity, there is a representative firm with productivity $\tilde{\phi}_{jki}$. The average productivity $\tilde{\phi}_{jki}$ summarizes all the information concerning each activity, and allows to aggregate variables. One key difference with respect to Melitz [2003] is that in that case, the average productivity for the whole economy depends only on domestic firms and thus has a closed-form solution. In our paper, the average productivity of a country will be given depend on domestic firms but also on foreign firms producing domestically. Then, aggregate variables for the whole economy will depend not only on the domestic mass of firms but also on the mass of firms from the rest of the countries.

The revenue at any productivity level, for domestic production $r_{iii}(\phi)$ with respect to the average revenue is given by

$$\frac{r(\tilde{\phi}_{iii})}{r_{iii}(\phi)} = \frac{E_i^1 P_i^{\sigma-1} \left(\frac{\rho\tilde{\phi}_{iii}}{w_i}\right)^{\sigma-1}}{E_i^1 P_i^{\sigma-1} \left(\frac{\rho\phi}{w_i}\right)^{\sigma-1}} \Rightarrow r(\tilde{\phi}_{iii}) = \left(\frac{\tilde{\phi}_{iii}}{\phi}\right)^{\sigma-1} r_{iii}(\phi) \qquad (19)$$

And similarly for the remaining activities,

$$\text{Exporting to country } k \Rightarrow r(\tilde{\phi}_{kii}) = \left(\frac{\tilde{\phi}_{kii}}{\phi}\right)^{\sigma-1} r_{kii}(\phi)$$

$$\text{DMP in country } k \Rightarrow r(\tilde{\phi}_{kki}) = \left(\frac{\tilde{\phi}_{kki}}{\phi}\right)^{\sigma-1} r_{kki}(\phi)$$

$$\text{BMP in } k \text{ to sell in } j \Rightarrow r(\tilde{\phi}_{jki}) = \left(\frac{\tilde{\phi}_{jki}}{\phi}\right)^{\sigma-1} r_{jki}(\phi)$$

Appendix A.4 shows that the distribution of revenues from each activity has a closed-form solution and is Pareto with shape parameter $\frac{\alpha}{\sigma-1}$ and scale parameter $r(\phi_{jki}^*)$.

4.5 Average Profits

Replacing (18) in the profit equations, we can calculate average profits in terms of average productivities. In the case that each individual activity makes zero profit at the cutoff level, we can obtain an analytical expression for the average profit in

each activity as:

$$\text{Selling Domestically} \Rightarrow \bar{\pi}_{iii} = \kappa_i^D w_i \left[\left(\frac{\tilde{\phi}_{iii}}{\phi_{iii}^*} \right)^{\sigma-1} - 1 \right]$$

$$\text{Exporting from the home country} \Rightarrow \bar{\pi}_{kii} = \kappa_i^X w_i \left[\left(\frac{\tilde{\phi}_{kii}}{\phi_{kii}^*} \right)^{\sigma-1} - 1 \right]$$

$$\text{DMP in country } k \Rightarrow \bar{\pi}_{kki} = \kappa_k^{MP} w_k \left[\left(\frac{\tilde{\phi}_{kki}}{\phi_{kki}^*} \right)^{\sigma-1} - 1 \right]$$

$$\text{BMP in } k \text{ to sell in } j \Rightarrow \bar{\pi}_{jki} = \kappa_k^X w_k \left[\left(\frac{\tilde{\phi}_{jki}}{\phi_{jki}^*} \right)^{\sigma-1} - 1 \right]$$

If, on the other hand, the profit at the cutoff level is not zero, due to lossess in the domestic market compensated by profits in foreign markets, then the average profit for that activity is

$$\bar{\pi}_i = \bar{\pi}_{iii} + \sum_{k \neq i} \theta_{kii} \bar{\pi}_{kii} + \sum_{k \neq i} \theta_{kki} \bar{\pi}_{kki} + \sum_{k \neq j} \sum_{k \neq i} \theta_{jki} \bar{\pi}_{jki} \, . \tag{20}$$

Notice that from Equation 41, profits are a function of aggregate expenditures E_i^1, which depends on population size. Thus, given the same variable costs and MP efficiency, multinationals prefer to set up factories in larger countries.

4.6 Mass of Firms

Let M_i^e to be the total mass of firms taking a productivity draw in country i, and $M = \theta_{iii} M_i^e$ the mass of firms that operate, where θ_{iii} is the probability of successful entry. The total mass of firms performing each of the other activities is obtained by multiplying the mass of firms operating, M_i, by the conditional probability of performing the activity $M_{jki} = \theta_{jki} M_i$. Notice that this is different from Melitz [2003] where the mass of firms is a function of aggregate revenue and average revenue, because in our case there are also foreign profits involved. Therefore, aggregate revenue needs not equal aggregate expenditure..

4.6.1 Aggregation

In this subsection we express the aggregate price and GDP of country i in terms of weighted average productivities. First, define M_i^P as the mass of firms producing

in country i and M_i^S as the mass of firms selling goods to country i. Then,

$$
\begin{aligned}
M_i^P &= M_i + \sum_{k \neq i} M_{iik} + \sum_{k \neq i} \sum_{i \neq j} M_{jik} \, , \\
M_i^S &= M_i + \sum_{k \neq i} M_{iik} + \sum_{k \neq j} \sum_{i \neq j} M_{ijk} \, .
\end{aligned}
\tag{21}
$$

In Appendix A.3 we show that, denoting the weighted average productivties of each mass as $\tilde{\phi}_i^P$ and $\tilde{\phi}_i^S$, aggregate price and aggregate production in the differentiated good sector in country i can be expressed as

$$
P_i = (M_i^S)^{\frac{1}{1-\sigma}} p(\tilde{\phi}_i^s) = (M_i^s)^{\frac{1}{1-\sigma}} \frac{w_i}{\rho \tilde{\phi}_i^s} \, ,
\tag{22}
$$

$$
GDP_i = M_i^P E_i \left(\frac{P_i \rho \tilde{\phi}_i^p}{w_i} \right)^{\sigma - 1}
\tag{23}
$$

4.7 Trade and Multinational Production

Now we express aggregate variables in terms of the average productivities. Exports are all the sales to foreign countries from firms (either domestic or foreign) producing in the domestic country. The expression for total exports in the differentiated good sector is

$$
\text{Exports}_i = X_i = \underbrace{\sum_{k \neq i} M_{kii} r_{kii}(\tilde{\phi}_{kii})}_{\text{Exports by Domestic Firms}} + \underbrace{\sum_{k \neq i} \sum_{k \neq j} M_{jik} r_{jik}(\tilde{\phi}_{jik})}_{\text{Exports by Foreign Firms}} \, .
$$

Similarly, imports in the differentiated good sector are all the goods consumed domestically and produced in a foreign country. Thus, total imports are given by:

$$
\text{Imports}_i = IM_i = \sum_{k \neq i} M_{ikk} r_{ikk}(\tilde{\phi}_{ikk}) + \sum_{k \neq i} \sum_{k \neq j} M_{ijk} r_{ijk}(\tilde{\phi}_{ijk}) \, .
$$

The capital account is composed of the difference between the profits of domestic firms producing abroad and the profits of foreign firms producing in the domestic country,

$$
\text{Capital Account}_i = \sum_{k} \sum_{j \neq i} M_{kji} \bar{\pi}_{kji} - \sum_{k} \sum_{j \neq i} M_{kij} \bar{\pi}_{kij} \, .
$$

The Trade Balance (TB) includes both the endowment of the homogeneous good, z_i, and the exports and imports in the differentiated sector from above. Thus,

$TB_i = (z_i - q_{i0}) + X_i - IM_i$, where $(z_i - q_{i0})$ is net exports of the homogeneous good. The Current Account (CA) is the trade balance plus the capital account balance,

$$\text{CA}_i = (z_i - q_{i0}) + X_i - IM_i + \sum_k \sum_{j \neq i} M_{kji} \bar{\pi}_{kji} - \sum_k \sum_{j \neq i} M_{kij} \bar{\pi}_{kij} \quad (24)$$

4.8 Equilibrium

The Zero Cutoff Profit Condition (ZCPC) establishes the cutoffs in each activity, steaming from Equation (20). The free entry condition (FEC) establishes that an entering firm's net value, v_i^e, should be equal to zero. Therefore,

$$v_i^e = \theta_{iii} \bar{\pi}_i - \kappa_i^e w_i = 0 \quad (25)$$

meaning that the average value of a firm producing in country i times the probability of successful entry, θ_{iii}, should be equal to the entry cost. The probability of a successful draw, θ_{iii} is in Equation 17 and is a function of the cutoffs. We define the definition of equilibrium below.

Definition: Given $\{z_{i0}, \tau_{ij}, \gamma_{ij}, \kappa_i^e, \kappa_i^d, \kappa_i^x, \kappa_i^{MP}, g_i(\phi), L_i\}_{i,j=1}^N$ a *multinational production equilibrium* is a set of wages $\{w_i\}_{i=1}^N$, price indices, $\{P_i\}_{i=1}^N$, income, $\{GNP_i\}_{i=1}^N$, masses of firms $\{M_i\}_{i=1}^N$, masses of entrants, $\{M_i^e\}_{i=1}^N$, and allocations and prices $\{q_{jki}(\phi), p_{jki}(\phi)\}_{i,j,k=1}^N$ such that:

1. Consumers maximize utility: given prices and aggregate expenditure $q_{jki}(\phi)$ and q_{io}) satisfy (8) and (9), in all countries,

2. Firms maximize profits: Equation (30) solves (10) in all countries.

3. P_i satisfies Equation (28)

4. Labor markets clear.

5. The Free Entry Condition: $v_i^e = 0$ (see Equation (25)) determines entry, and the Zero Cutoff Profit Condition determines the cutoffs.

6. The Current Account balance is zero (see Equation (24)).

7. The mass of firms producing in each country is $M_i = \theta_{iii} M_i^e$

8. World demand of the homogeneous good is equal to world supply:

$$\sum_i z_i = \sum_i q_{i0}$$

.

Notice that this is a system of $3N + 1$ variables, where N is the number of countries. We need N cutoffs ($\phi_{iii}^{*}, \forall i$), N masses of firms ($M_i \forall i$), N wages (w_i), and one price (P_0). Normalizing the price of the homogeneous good to one, we end up with $3N$ endogenous variables. Cutoffs are obtained from the ZPCP, and the masses are obtained from the FEC. Wages are obtained from labor market equilibrium. All remaining variables are a function of these. Appendix A.5 explains the algorithm.

5   Calibration and Results

The model is calibrated separately for South America and Europe. For South America, we include Argentina, Brazil, Chile, and Uruguay. For Europe, we include four members of the European Union: France, Italy, the Netherlands, and the United Kingdom. In both cases, there is also a fifth country which stands for the rest of the world (RoW). We use data on bilateral trade flows from Waugh [2010], bilateral FDI flows and firm status (domestic, foreign, exporters, and non-exporters), GDP per capita, manufacturing trade deficit, and labor force size. We use, when possible, the year 199, after the Mercosur trade agreement was signed. The rest of the world includes all countries in Waugh [2010] that are not directly in the calibration.

Calibrated parameters are in Table 2. Country size is calibrated using data from UNCTAD, normalizing labor force in Uruguay to one. The substitutability between varieties, σ generates a markup of 20%. The parameters of the Pareto distribution, assumed identical across countries, is a subject of debate. We choose to follow Arkolakis and Muendler [2010], that estimate this parameter for Brazilian data, and is also the median among other estimates.[2]

The fixed entry cost, κ_i^e, is set to match the GDP per capita in each country relative to the RoW. Entry costs are lower in Europe to match its higher GDP per capita with respect to the RoW. The fixed operating cost, κ_i^d, is set so that each country's smallest firm demands 10 workers (See Appendix A.2). The domestic productivity cutoff depends only on σ and κ_i^d, and thus it will not vary across countries.

The fixed cost of exporting, κ_i^x, is set to match the proportion of firms exporting as a fraction of the total number of operating firms. The fixed cost of doing MP, κ_i^{MP}, is calibrated to match the proportion of foreign firms in a given country. These statistics are obtained from the World Bank Enterprise Survey for South America and the OECD Stan for Europe.

---

[2]See Chaney [2008], Ramondo and Rappoport [2010], Breinlich and Cuñat [2010], and Arkolakis et al. [2018].

Panel A

	L_i	κ^e	κ^d	κ^x	κ^{MP}	z	σ	α
Argentina	9.47	0.09	1.67	0.34	11.77	0.13	6.00	1.21
Brazil	48.94	1.95	1.67	1.15	2.07	0.35	6.00	1.21
Chile	3.69	0.13	1.67	2.05	19.07	0.04	6.00	1.21
Uruguay	1.00	0.07	1.67	0.82	9.17	0.01	6.00	1.21
Rest of the World	1582.5	3.00	1.67	1.00	2.67	12.66	6.00	1.21

Panel B

	L_i	κ^e	κ^d	κ^x	κ^{MP}	z	σ	α
France	16.8	3.3e-6	1.67	0.89	5.42	0.78	6.00	1.21
United Kingdom	18.7	3.0e-6	1.67	1.50	10.07	0.90	6.00	1.21
Italy	14.9	1.0e-6	1.67	1.25	9.87	0.73	. 6.00	1.21
Netherlands	4.9	1.0e-6	1.67	3.32	10.97	0.25	6.00	1.21
Rest of the World	1567.3	1.00	1.67	1.00	2.67	15.67	6.00	1.21

Table 2: Calibrated Parameters

The iceberg costs of exporting, τ_{ji}, are set to match the trade volumes of manufactures (imports plus exports) over total domestic absorption (output minus exports plus imports). These targets are constructed from Waugh [2010]. The productivity shifter, γ_{ji}, is calibrated to match the proportion of sales from foreign firms in the domestic country. Using data from WBES, we obtain the participation of foreign sales on total sales. Given that it does not provide information on the origin, we compute the composition of FDI stock in manufacture to impute these values, obtained from UNCTAD Foreign Direct Investment Profile for South America and OECD Stan for Europe. The average calibrated parameters are in Figure 2. These parameters are destination-dependent; the complete list of calibrated parameters is in Appendix A.6. For Europe, trade-to-absorption is much higher than in South America. In order to match the higher ratio, the model requires smaller trade costs in the EU. Finally, the endowment of the homogeneous good, z_i, is calibrated to match the trade deficit in the manufacturing sector.

The efficiency parameter $\frac{1}{\gamma}$ is lower for South America than Europe, and foreign firms are much less productive operating in South America than in Europe. The average value of γ is 2.15 in South America and 1.3 in Europe. The fact γ is lower in Europe is mainly driven by the productivity of firms from the RoW. Firms from the RoW operating in Europe are three times more efficient than in their domestic countries. See Table 7.
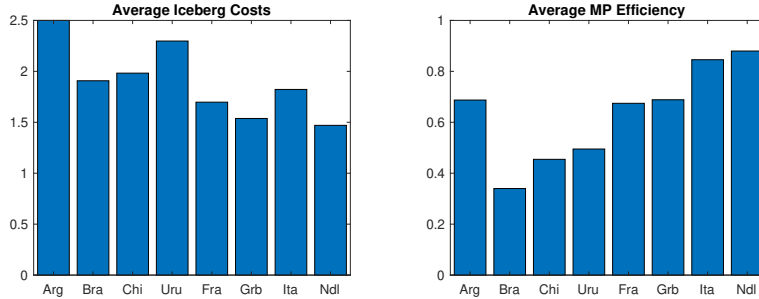
Figure 2: Average iceberg costs and Average MP efficiency ($\frac{1}{\gamma}$) at each destination. See Table 7 in Appendix A.6.

5.1 Model Fit

Table 3 presents the model fit for Trade volume and Foreign sales. Note that Argentina and Brazil, the two largest countries in South America, show lower ratios of Trade-to-Absorption: 35.8% and 22.8%. Chile and Uruguay, the smallest countries, show much higher ratios: 59.4% and 58.3%. In order to match the large proportion of domestic firms exporting in Argentina, the model requires small fixed costs of exporting for this country. The importance of the RoW as a trade partner is also shown in the calibrated parameters. The participation of the RoW in trade goes from 51% for Uruguay to 86% for Chile.

Figure 3 shows that the model also matches the trade balance over absorption in the manufacturing sector, the percentages of foreign and exporting firms, and GDP per capita. The GDP per capita of the RoW is normalized to 1. The model performs well in matching the selected targets.

The baseline model is consistent with cross-country evidence on bilateral trade flows and multinational production for the selected countries. South America faces higher trade barriers than Europe, and these trade barriers vary with country size among regions. South American countries cannot attract as much MP as European countries because the productivity of multinationals operating in South America is lower than that of multinationals operating in Europe.

6 Experiments

We use the model to perform a series of counterfactual experiments. First, we investigate the gains from openness by comparing output in autarky to the calibrated model. Second, we focus on the determinants of the variable costs: iceberg
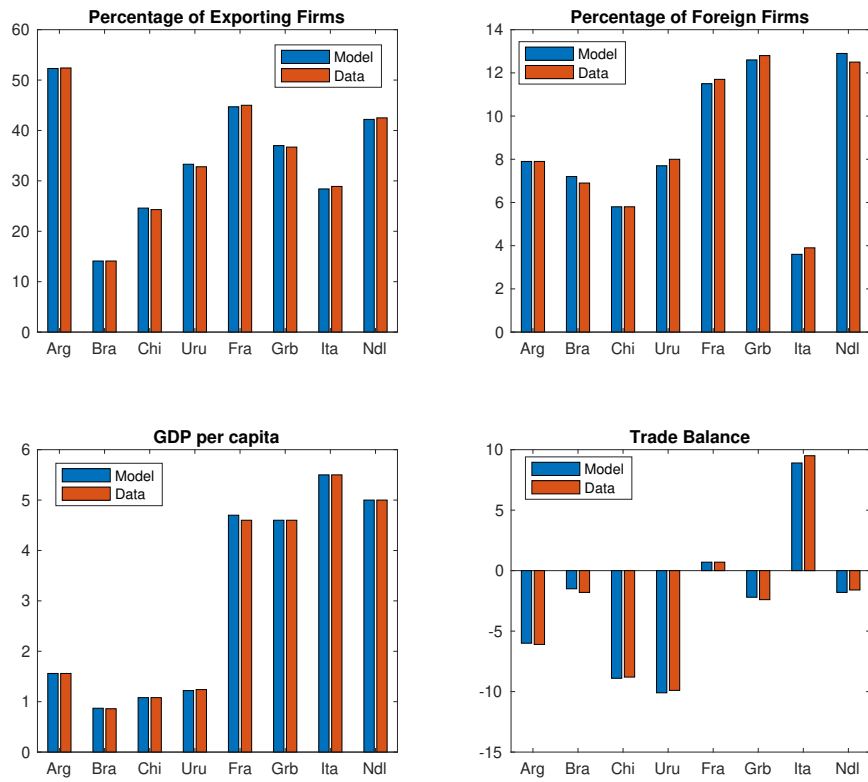
Figure 3: Model fit, aggregate variables.

South America

	Trade as % of Absorption									
	ARG		BRA		CHL		URU		RoW	
	Data	Model	Data	Model	Data	Model	Data	Model	Data	Model
ARG	-	-	2.9	3.3	3.5	3.8	9.5	9.8	0.2	0.2
BRA	9.6	9.1	-	-	4.2	4.4	17.2	16.8	0.5	0.5
CHL	1.5	1.1	0.5	0.4	-	-	1.8	1.5	0.2	0.1
URU	0.8	0.9	0.5	0.5	0.4	0.5	-	-	0.0	0.0
RoW	24.0	24.2	18.9	18.9	51.3	51.5	29.9	30.1	-	-
Total	35.8	35.1	22.8	23.2	59.4	60.1	58.3	58.2	0.9	0.9

	Foreign Sales as % of Total Sales								
Arg	-	-	0.1%	0.1%	0.7%	0.6%	0.0%	0%
Bra	1.4%	1.5%	-	-	0.3%	0.3%	0.0%	0%
Chi	1.4%	1.4%	0.1%	0.1%	-	-	0.0%	0%
Uru	1.1%	1.0%	0.2%	0.2 %	0.3%	0.3%	-	-
RW	31.9%	31.7%	7.8%	8.0%	32.5%	32.3%	29.7%	30.7%

European Union

	Trade as % of Absorption									
	FRA		GBR		ITA		NDL		RoW	
	Data	Model	Data	Model	Data	Model	Data	Model	Data	Model
FRA	-	-	8.5	6.3	8.6	7.5	13.8	11.2	2.3	1.7
GBR	6.7	7.2	-	-	4.4	5.1	18.6	18.9	2.6	2.8
ITA	7.6	7.2	5.0	4.3	-	-	9.7	10.8	1.7	1.3
NDL	3.4	3.6	5.8	5.3	2.7	3.6	-	-	1.2	1.2
RoW	34.2	34.7	49.5	49.7	28.4	28.0	76.2	77.2	-	-
Total	51.9	52.7	68.9	65.6	44.1	44.2	118.4	118.1	7.7	7.1

	Foreign Sales as % of Total Sales								
FRA	-	-	2.5%	2.1%	2.5 %	2.6%	2.9%	2.6%
GBR	3.0%	3.4%	-	-	1.6%	1.9%	4.9%	5.3%
ITA	1.6%	2.1%	1.2%	1.3%	-	-	0.0%	0.0%
NDL	1.4%	1.4%	2.1 %	1.6 %	1.3%	1.2	-	-
RoW	20.1%	20.3%	38.9%	37.8%	13.2%	12.7%	35.0%	35.1%

Table 3: Model Fit, trade composition.

costs and country efficiency, and the potential gains in real GDP. We disentangle
the role of MP, BMP, and trade in countries of different sizes and the gains from

integration in the two regions under study.

6.1 Gains from Openness

To study the gains from openness we set the world in autarky, i.e., $\gamma_{ij} = \tau_{ij} = \infty$.

Panel A

	Changes in %			
	Autarky with BMP		Autarky without BMP	
	Real GDP	Real GNP	Real GDP	Real GNP
South-America	-5.3	-3.4	-5.0	-3.5
Argentina	-9.5	-4.9	-9.0	-5.2
Brazil	-3.6	-2.5	-3.5	-2.5
Chile	-11.9	-8.7	-10.9	-8.9
Uruguay	-12.1	-10.8	-10.9	-10.4

Panel B

	Changes in %			
	Autarky with BMP		Autarky without BMP	
	Real GDP	Real GNP	Real GDP	Real GNP
Europe	-10.5	-7.3	-9.3	-7.3
France	-9.1	-6.4	-8.3	-6.3
UK	-13.4	-8.8	-11.9	-9.0
Italy	-5.6	-3.5	-5.1	-3.6
Netherlands	-20.3	-17.1	-17.0	-16.5

Table 4: Experiment Results, closing the economies

The first two columns of Table 4 present the changes in real manufacturing GDP and GNP using the calibrated model economies as a benchmark. Panel A presents the results for South America and Panel B for Europe. Losses of moving to autarky in Europe are much larger than in South America (10.5% versus 5.3% of real GDP), which indicates that Europe benefits much more from openness than South America. This is because, in South America, trade costs are higher, and the efficiency of foreign firms is lower than in Europe. Small countries lose more than large countries in both regions. Compared to South America, the higher degree of openness in Europe results in larger differences between the country that loses the most and the country that loses the least. In Europe, autarky would imply a loss of 20.3% of real GDP for the Netherlands, whereas in South America,f Uruguay loses 12.1%.

In order to assess the role of BMP, we first calculate the changes in real manufacturing GDP and GNP using as a benchmark an economy where BMP is not

allowed. These results are presented in the last two columns of Table 4. Comparing the results of the third column to the Compared to South America, the higher degree of openness in Europe results in larger differences between the country that loses the most and the country that loses the least.first column, we can see that BMP is more important in small countries than in large countries, and that European countries benefit more from BMP. In a world without BMP, the Netherlands would lose 3.3p.p, whereas Uruguay would only lose 1.2p.p. High trade barriers affect the exports of domestic firms and the exports of foreign firms, and as a result, the ability of small countries to attract multinational firms.

We perform three experiments to disentangle the effect of each channel in the Gains from trade. The results are in Table 5. We compare the losses of going to autarky in a world without BMP and the losses of going to autarky in a world without MP and express it as a percentage of total gains from trade.

The DMP channel accounts for 42.4% of the total gains from openness in Italy but only 15.8% in the Netherlands. DMP is more important in explaining the gains from MP in a large country than in a small one. In South America, both Brazil and Uruguay present similar gains through the DMP channel, 33.4%, and 32.4%. These gains differ because the efficiency of multinational firms operating in Brazil is low, and therefore MP is not a cheap way of overcoming trade barriers. However, in Argentina, the DMP channel accounts for 50.1% of the total gains from trade.

The BMP channel, where we allow DMP but no BMP, is in the second column of Table 5. The contribution is larger for small countries than large countries and higher in Europe than in South America. BMP channel accounts for 11.1% in Uruguay and 19.6% in the Netherlands of the total gains from openness. In Brazil and Italy, the contribution of this channel is 3.5% and 9.7%, respectively.

Finally, in the last column of Table 5, we report the gains from openness from both channels. This number shows the aggregate contribution of MP in explaining the gains from openness – the remaining percentage belonging to exporting. Argentina and Italy are the countries that benefit the most in South America and Europe. Brazil and the Netherlands benefit the least in each region, the largest and smallest countries in each region. Since in Brazil the efficiency of multinational firms is low, the role played by MP is lower. Moreover, given that South America as a region is relatively closed, then the gains from trade are not as large, which increases the importance of MP in explaining the gains from openness.

To sum up, if countries face relatively low trade costs and high efficiency of foreign firms, large countries benefit more from MP as a whole, with small countries benefitting more from BMP. Moreover, in more integrated countries with high ef-

Panel A

	Contribution to total gains from openness		
	DMP channel	BMP channel	BMP+DMP
Argentina	50.1%	5.8%	55.9%
Brazil	33.4%	3.5%	36.9%
Chile	31.4%	9.9%	41.3%
Uruguay	32.4%	11.1%	43.5%

Panel B

	Contribution to total gains from openness		
	DMP channel	BMP channel	BMP+DMP
France	38.6%	9.2%	47.9%
UK	37.7%	12.7%	50.5%
Italy	42.4%	9.7%	52.1%
Netherlands	15.8%	19.6%	35.4%

Table 5: The effects of MP and BMP

ficiency, big countries can benefit from MP more than smaller countries, whereas this is not necessarily the case in less integrated countries.

Discussion: The Cost of Brexit. The model can be used to assess the impact of Brexit. We run the simulations closing the BMP channel for the UK, allowing firms to still locate in the UK to export to the rest of the world. The model predicts that the UK could lose up to 0.5% of its manufacturing GDP by exiting the European Union. Recently, Kierzenkowski et al. [2016] and Dhingra et al. [2015] have found that the GDP of the UK can fall around 3% if the UK does not reach a new trade agreement with Europe, which means that the BMP could be a significant channel to understand the total Brexit loses.

6.2 Reducing trade costs and improving efficiency

In this section we study the potential gains in South America of changes in variable costs: the degree of openness and multinational efficiency. For the first task, we reduce the average trade costs for all countries in South America to the average level in Europe. For the second task, in addition to reducing trade costs, we explore the effect of an increase in the productivity of multinational firms.

Panel A of Table 6 presents the result of reducing trade costs in South America to the average level in Europe. All countries gain by reducing trade costs, but the smallest country, Uruguay, gains significantly more. The gains in Uruguay are

29.9% of real manufacturing GDP, while in Brazil, the largest country, they are just 4%. Consistent with Eaton and Kortum [2002], we find that the gains from reducing trade costs are larger than the losses of going to autarky.

Besides the lower trade costs, the second experiment increases MP efficiency by 20%, allowing us to assess the potential gains that arise from the interaction of trade and MP.

Panel B of Table 6 presents the results of this experiment. There is a large gain in real manufacturing GDP in all countries. However, since multinational firms send their profits back, the increase is not generally reflected in real manufacturing GNP, except for Uruguay. The third and fourth columns show the results of shutting down the BMP channel. Compared to the numbers in Panel A, we see that Uruguay is the country with the smallest additional increase in manufacturing real GDP (0.7 p.p.). In comparison, the remaining countries show increases that go from 1.9 p.p. to 4 p.p. This result indicates that to benefit from increases in the efficiency of multinationals, BMP is crucial for Uruguay. Otherwise, the gains would not be larger than the ones it would get by only reducing trade costs.

Panel C of Table 6 presents the result of increasing the efficiency to multinationals operating only in Uruguay. Changes in real manufacturing GDP for the rest of the countries are the same as when only reducing trade costs. At the same time, in Uruguay, it increases by an extra 12p.p. Uruguay gains more if efficiency improves only domestically and not in the whole region because it attracts a larger number of multinationals. If we shut down BMP, the additional increase with respect to the benchmark is only 2.3p.p. Without the possibility of serving third countries, Uruguay does not become an attractive location for multinational firms because its domestic market is small.

The previous experiments reflect the importance of BMP for a small country. In the absence of BMP, the gains in real manufacturing GDP of reducing trade barriers decrease for all countries, but they decrease significantly more for Uruguay. When trade costs are reduced, small countries can attract more foreign firms who will locate there to export to the rest of the countries, explaining the importance of BMP. This result indicates that for small countries, increasing productivity allows attracting multinationals to serve neighbors.

Panel A

| | Changes (in %) | | | |
| | Same MP costs | | Same MP Costs-No BMP | |
	Real GDP	Real GNP	Real GDP	Real GNP
South-America	6.2	6.1	5.3	6.0
Argentina	11.2	11.1	9.0	11.7
Brazil	4.0	4.0	3.7	4.0
Chile	13.1	12.4	9.4	12.4
Uruguay	29.9	29.1	23.7	27.1

Panel B

| | Changes (in %) | | | |
| | Improve 20% efficiency | | Improve 20% efficiency- No BMP | |
	Real GDP	Real GNP	Real GDP	Real GNP
South-America	9.7	6.6	7.6	6.9
Argentina	17.7	11.7	13.0	13.0
Brazil	6.3	4.3	5.6	4.5
Chile	21.4	14.3	12.4	14.3
Uruguay	38.3	36.6	24.4	30.7

Panel C

| | Changes (in %) | | | |
| | Improve 20% efficiency only in Uruguay | | Improve 20% efficiency only in Uruguay- No BMP | |
	Real GDP	Real GNP	Real GDP	Real GNP
South-America	6.3	6.1	5.3	6.2
Argentina	11.1	11.1	9.0	11.7
Brazil	4.0	4.0	3.7	4.0
Chile	13.1	12.4	9.4	12.4
Uruguay	41.8	29.3	26.0	28.1

Table 6: Experiment Results, reducing Costs. All experiments use the average trade costs in Europe ($\tau = 1.64$)

Discussion of Assumption 1

Assumption 1 states that a firm can serve one market from all possible locations, treating each activity independently and thus simplifying the problem. Notice that the assumption reduces the degree of competition between countries to attract MP. The decrease in competition also reduces the importance of the efficiency of multinationals operating in a country. Thus, the gains we obtained from reducing trade barriers and improving efficiency will be higher without the assumption. This channel is particularly important for BMP in small countries:

since the firm wants to serve all countries from the cheapest location, the most efficient market will attract more MP. Small countries will have a larger incentive to attract BMP. Thus, the importance of BMP for a small country might be underestimated, and the results are a lower bound for the importance of MP and BMP.

7   Conclusion

We construct a heterogeneous firms model of trade with asymmetric countries, MP, and BMP to quantitatively study the effects of trade barriers and country size in the location decision of multinational firms and thus in the gains from trade.

We first show theoretically that larger countries can attract more multinational firms because to serve a large domestic market. However, allowing for multinationals to bridge to third party countries allows small countries to benefit from trade relatively more, since multinationals can settle in them and serve other markets from there. Assessing country size is essential to understand the location decisions of multinationals and the benefits from integration.

DMP is relatively more important for large countries that can attract firms due to its market size, whereas BMP is crucial for small countries. Thus, it is critical to take advantage of trade liberalization and efficiency improvements to attract MP. BMP explains up to 20% of the gains from openness in the Netherlands while only 10% in Uruguay.

Increasing integration in South America to the level in Europe would increase Uruguay's real manufacturing GDP by 30%. Without BMP, this is reduced by 6p.p. An increase in the efficiency of multinationals operating in Uruguay of 20% would imply an increase in real manufacturing GDP of 41.8%. BMP explains almost all the additional increase in manufacturing real GDP.

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A   Appendix

A.1   Proof

We need to characterize the number of entrants under the system of Equations 2 - 6. The demand level is given by

$$B = \frac{1-\alpha}{\alpha^{1-\epsilon}} \frac{\beta E^i}{\int_0^{n_i} p^i(\nu)^{1-\epsilon} d\nu} = \frac{1-\alpha}{\alpha^{1-\epsilon}} \frac{\beta L^i}{\int_0^{n_i} p^i(\nu)^{1-\epsilon} d\nu} \rightarrow \int_0^{n_i} p^i(\nu)^{1-\epsilon} d\nu = \frac{1-\alpha}{\alpha^{1-\epsilon}} \frac{\beta L^i}{B}$$

where E^i is aggregate spending

$$\int_0^{n_i} p^i(\nu)^{1-\epsilon} d\nu \quad = n_E^i \int_0^{a_D} \left(\frac{a}{\alpha}\right)^{1-\epsilon} dG(a) + \sum_{j \neq i} n_E^j \int_0^{a_M} \left(\frac{a}{\alpha}\right)^{1-\epsilon} dG(a) +$$

$$+ \sum_{j \neq i} n_E^j \int_0^{a_X} \left(\frac{\tau a}{\alpha}\right)^{1-\epsilon} dG(a) + \sum_j n_j \sum_k \int_0^{a_B} \left(\frac{\tau \gamma a}{\alpha}\right)^{1-\epsilon} dG(a)$$

We can rewrite this as

$$\int_0^{n_i} p^i(\nu)^{1-\epsilon} d\nu = \frac{v_D n_E^i + v_{MX} \sum_{j \neq i} n_E^j}{\alpha^{1-\epsilon}}$$

where $v_D = V(a_D)$ and $v_{MX} = \tau^{1-\epsilon} V(a_M) + V(a_X) + \tau^{1-\epsilon} \gamma V(a_B) N(N-1)/2$. This can be put in Matrix form to solve for n_E^j.

$$
\begin{bmatrix}
v_D & v_{MX} & \cdots & v_{MX} \\
v_{MX} & v_D & \cdots & v_{MX} \\
\vdots & \vdots & \ddots & \vdots \\
v_{MX} & v_{MX} & \cdots & v_D
\end{bmatrix}
\begin{bmatrix}
n_E^1 \\
n_E^2 \\
. \\
. \\
n_E^N
\end{bmatrix}
= \frac{(1-\alpha)\beta}{B}
\begin{bmatrix}
L^1 \\
L^2 \\
. \\
. \\
L^N
\end{bmatrix}
\tag{26}
$$

Assume for now that the $n_E^i > 0 \forall i$, then the solution to the system is

$$n_E^i = \frac{(1-\alpha)\beta}{B \det(V)} \left\{ ((N-1)v_{MX} + v_D) L_i - v_{MX} \sum_j L^j \right\} \tag{27}$$

where V is the matrix of coefficients. The number of entrants is positive if $\frac{L_i}{\sum_j L_j} > \frac{v_{MX}}{(N-1)v_{MX} + v_D}$. Moreover, $\det(V) > 0$ since $v_D > v_{MX} > 0$. The results follow from taking partial derivatives of Equation 27.

A.2 Labor for the smaller firm operating

The amount of labor demanded by the smaller firm is:[3]

$$
\begin{aligned}
\ell(\phi_{iii}^*) &= \frac{q(\phi_{iii}^*)}{\phi_{iii}^*} + \kappa_i^d \\
q(\phi_{iii}^*) &= \frac{r(\phi_{iii}^*)}{p(\phi_{iii}^*)} \\
\text{from (35)} \rightarrow r(\phi_{iii}^*) &= \sigma w_i \kappa_i^d \\
\text{and from equation (30)} \rightarrow p(\phi_{iii}^*) &= \frac{\sigma}{\sigma-1} \frac{w_i}{\phi_{iii}^*} \\
\text{then} \rightarrow q(\phi_{iii}^*) &= (\sigma-1)\kappa_i^d \phi_{iii}^* \\
\ell(\phi_{iii}^*) &= \sigma \kappa_i^d
\end{aligned}
$$

A.3 Aggregation

First we present the definition of Aggregate Price, and average productivities. We define aggregate price and GDP in country i as:

$$
\begin{aligned}
P_i &= \left[\int_{\phi_{iii}^*} (p_{iii}(\phi))^{1-\sigma} M_i \mu_i(\phi) d\phi + \sum_{k \neq i} \int_{\phi_{ikk}^*} (p_{ikk}(\phi))^{1-\sigma} M_k \mu_k(\phi) d\phi \right. \\
&\quad \left. + \sum_{k \neq i} \int_{\phi_{iik}^*} (p_{iik}(\phi))^{1-\sigma} M_k \mu_k(\phi) d\phi + \sum_{k \neq j} \sum_{k \neq i} \int_{\phi_{ikj}^*} (p_{ikj}(\phi))^{1-\sigma} M_j \mu_j(\phi) d\phi \right]^{\frac{1}{1-\sigma}},
\end{aligned} \tag{28}
$$

$$
\begin{aligned}
GDP_i &= \int_{\phi_{iii}^*} r_{iii}(\phi) M_i \mu_i d\phi + \sum_{k \neq i} \int_{\phi_{kii}^*} r_{kii}(\phi) M_i \mu_i d\phi + \sum_{k \neq i} \int_{\phi_{iik}^*} r_{iik}(\phi) M_k \mu_k d\phi \\
&\quad + \sum_{k \neq j} \sum_{k \neq i} \int_{\phi_{kij}^*} r_{kij}(\phi) M_j \mu_j d\phi .
\end{aligned} \tag{29}
$$

where

$$
p_{jki}(\omega) = \frac{w_k \gamma_{ki} \tau_{jk}}{\rho \phi} \tag{30}
$$

with $\tau_{ii} = \gamma_{ii} = 1$ and revenues for each activity are (using expression (9) and

---

[3]This is true as long as the domestic cutoff is the lowest cutoff.

(30)) are r_{kji},

$$\text{Selling Domestically} \Rightarrow r_{iii}(\phi) = E_i^1 P_i^{\sigma-1}\left(\frac{\rho\phi}{w_i}\right)^{\sigma-1}$$

$$\text{Exporting from the home country} \Rightarrow r_{kii}(\phi) = E_k^1 P_k^{\sigma-1}\left(\frac{\rho\phi}{w_i\tau_{ki}}\right)^{\sigma-1} \quad (31)$$

$$\text{Doing DMP in country } k \Rightarrow r_{kki}(\phi) = E_k^1 P_k^{\sigma-1}\left(\frac{\rho\phi}{w_k\gamma_{ki}}\right)^{\sigma-1}$$

$$\text{Doing BMP in } k \text{ to sell in } j \Rightarrow r_{jki}(\phi) = E_j^1 P_j^{\sigma-1}\left(\frac{\rho\phi}{w_k\gamma_{ki}\tau_{jk}}\right)^{\sigma-1}$$

Let us define the average productivity of firms performing each activity:

$$\tilde{\phi}_{iii} = \left[\int_{\phi_{iii}^*}^{\infty}\phi^{\sigma-1}\mu_i d\phi\right]^{\frac{1}{1-\sigma}}$$

$$\tilde{\phi}_{kii} = \left[\int_{\phi_{kii}^*}^{\infty}\phi^{\sigma-1}\mu_i d\phi\right]^{\frac{1}{1-\sigma}}$$

$$\tilde{\phi}_{kki} = \left[\int_{\phi_{kki}^*}^{\infty}\phi^{\sigma-1}\mu_i d\phi\right]^{\frac{1}{1-\sigma}}$$

$$\tilde{\phi}_{jki} = \left[\int_{\phi_{jki}^*}^{\infty}\phi^{\sigma-1}\mu_i d\phi\right]^{\frac{1}{1-\sigma}}$$

We can also define the weighted average productivity of firms producing ($\tilde{\phi}_i^P$) and selling ($\tilde{\phi}_i^S$) as:

$$\tilde{\phi}_i^P = \left\{\frac{1}{M_i^P}\left[M_{iii}\tilde{\phi}_{iii}^{\sigma-1} + \sum_{k\neq i}M_{kii}\frac{E_k^1}{E_i^1}\left(\frac{P_k}{\tau_{ki}P_i}\right)^{\sigma-1}\tilde{\phi}_{kii}^{\sigma-1} + \sum_{k\neq i}M_{iik}\left(\frac{1}{\gamma_{ik}}\right)^{\sigma-1}\tilde{\phi}_{iik}^{\sigma-1}\right.\right.$$

$$\left.\left. + \sum_{k\neq i}\sum_{i\neq j}M_{jik}\frac{E_j^1}{E_i^1}\left(\frac{P_j}{\tau_{ji}\gamma_{ik}}\right)^{\sigma-1}\tilde{\phi}_{jik}^{\sigma-1}\right]\right\}^{\frac{1}{\sigma-1}}, \quad (32)$$

$$\tilde{\phi}_i^S = \left\{\frac{1}{M_i^S}\left[M_{iii}\tilde{\phi}_{iii}^{\sigma-1} + \sum_{k\neq i}M_{ikk}\left(\frac{w_k\tau_{ik}}{w_i}\right)^{1-\sigma}\tilde{\phi}_{ikk}^{\sigma-1} + \sum_{k\neq i}M_{iik}\gamma_{ik}^{1-\sigma}\tilde{\phi}_{iik}^{\sigma-1}\right.\right.$$

$$\left.\left. + \sum_{k\neq i}\sum_{i\neq j}M_{ijk}\left(\frac{\tau_{ij}\gamma_{jk}w_k}{w_i}\right)^{1-\sigma}\tilde{\phi}_{ijk}^{\sigma-1}\right]\right\}^{\frac{1}{\sigma-1}}. \quad (33)$$

Combining Equation 30 and Equation 28

$$
\begin{aligned}
P_i \;=\;& \left[\int_{\phi^*_{iii}} \left(\frac{w_i}{\rho\phi}\right)^{1-\sigma} M_i \mu_i(\phi)d\phi + \sum_{k\neq i} \int_{\phi^*_{ikk}} \left(\frac{w_k \tau_{ik}}{\rho\phi}\right)^{1-\sigma} M_k \mu_k(\phi)d\phi \right. \\
&\left. + \sum_{k\neq i} \int_{\phi^*_{iik}} \left(\frac{w_i \gamma_{ik}}{\rho\phi}\right)^{1-\sigma} M_k \mu_k(\phi)d\phi + \sum_{k\neq j}\sum_{k\neq i} \int_{\phi^*_{ikj}} \left(\frac{w_k \gamma_{kj}\tau_{ik}}{\rho\phi}\right)^{1-\sigma} M_j \mu_j(\phi)d\phi \right]^{\frac{1}{1-\sigma}}
\end{aligned}
$$

$$
\begin{aligned}
P_i \;=\;& \left[\left(\frac{w_i}{\rho}\right)^{1-\sigma} M_i \int_{\phi^*_{iii}} (\phi)^{\sigma-1}\,\mu_i(\phi)d\phi + \sum_{k\neq i} M_{ikk}\left(\frac{w_k \tau_{ik}}{\rho}\right)^{1-\sigma} \int_{\phi^*_{ikk}} (\phi)^{\sigma-1}\,\mu_k(\phi)d\phi \right. \\
&+ \sum_{k\neq i} M_{iik}\left(\frac{w_i \gamma_{ik}}{\rho}\right)^{1-\sigma} \int_{\phi^*_{iik}} (\phi)^{\sigma-1}\,\mu_k(\phi)d\phi \\
&\left. + \sum_{k\neq j}\sum_{k\neq i} M_{ikj}\left(\frac{w_k \gamma_{kj}\tau_{ik}}{\rho}\right)^{1-\sigma} \int_{\phi^*_{ikj}} (\phi)^{\sigma-1}\,\mu_j(\phi)d\phi \right]^{\frac{1}{1-\sigma}}
\end{aligned}
$$

We can replace the integral terms by each of the average productivities,

$$
\begin{aligned}
P_i \;=\;& \frac{w_i}{\rho}\left[M_{iii}\tilde{\phi}_{iii}^{1-\sigma} + \sum_{k\neq i} M_{ikk}\left(\frac{w_k \tau_{ik}}{w_i}\right)^{1-\sigma} \tilde{\phi}_{ikk}^{1-\sigma} \right. \\
&\left. + \sum_{k\neq i} M_{iik}\gamma_{ik}^{1-\sigma}\tilde{\phi}_{iik}^{1-\sigma} + \sum_{k\neq j}\sum_{k\neq i} M_{ikj}\left(\frac{w_k \gamma_{kj}\tau_{ik}}{w_i}\right)^{1-\sigma} \tilde{\phi}_{ikj}^{1-\sigma} \right]^{\frac{1}{1-\sigma}}
\end{aligned}
$$

Note that the term inside brackets is $\dfrac{(M^s_i)^{\frac{1}{\sigma-1}}}{\tilde{\phi}^s_i}$, and that $p(\tilde{\phi}^s_i) = \dfrac{w_i}{\rho\tilde{\phi}^s_i}$. Then

$$
P = (M^s_i)^{\frac{1}{1-\sigma}} p(\tilde{\phi}^s_i)
$$

In a similar way we can derive the equation for aggregate GDP using Equation 29

and Equation 31.

$$
\begin{aligned}
GDP_i &= E_i^1 P_i^{\sigma-1} \left(\frac{\rho}{w_i}\right)^{\sigma-1} M_i \int_{\phi_{iii}^*} \phi^{\sigma-1}\mu_i d\phi + \sum_{k\neq i} E_k^1 P_k^{\sigma-1} \left(\frac{\rho}{w_i\tau_{ki}}\right)^{\sigma-1} M_i \int_{\phi_{kii}^*} \phi^{\sigma-1}\mu_i d\phi \\
&+ \sum_{k\neq i} E_i^1 P_i^{\sigma-1} \left(\frac{\rho}{w_i\gamma_{ik}}\right)^{\sigma-1} M_k \int_{\phi_{iik}^*} \phi^{\sigma-1}\mu_k d\phi \\
&+ \sum_{k\neq j}\sum_{k\neq i} E_k^1 P_k^{\sigma-1} \left(\frac{\rho}{w_i\gamma_{ij}\tau_{ki}}\right)^{\sigma-1} M_j \int_{\phi_{kij}^*} \phi^{\sigma-1}\mu_j d\phi
\end{aligned}
$$

We can replace again the integral terms by the average productivities for each activity,

$$
\begin{aligned}
GDP_i &= E_i^1 P_i^{\sigma-1} \left(\frac{\rho}{w_i}\right)^{\sigma-1} \left[M_i \tilde{\phi}_{iii}^{1-\sigma} + \sum_{k\neq i} \frac{E_k^1}{E_i^1} \left(\frac{P_k}{P_i w_i \tau_{ki}}\right)^{\sigma-1} M_{kii}\tilde{\phi}_{kii}^{1-\sigma} \right. \\
&+ \left. \sum_{k\neq i} \left(\frac{1}{\gamma_{ik}}\right)^{\sigma-1} M_{iik}\tilde{\phi}_{iik}^{1-\sigma} \sum_{k\neq j}\sum_{k\neq i} \frac{E_k^1}{E_i^1} \left(\frac{P_k}{P_i\gamma_{ij}\tau_{ki}}\right)^{\sigma-1} M_j\tilde{\phi}_{kij}^{1-\sigma} \right]
\end{aligned}
$$

Note that the term in brackets is equal to $M_i^p \left(\tilde{\phi}_i^p\right)^{\sigma-1}$, then

$$
GDP_i = M_i^p E_i^1 P_i^{\sigma-1} \left(\frac{\rho}{w_i}\right)^{\sigma-1} \left(\tilde{\phi}_i^p\right)^{\sigma-1}
$$

and as $r_{iii}(\tilde{\phi}_i^p) = E_i^1 P_i^{\sigma-1} \left(\frac{\rho\tilde{\phi}_i^p}{w_i}\right)^{\sigma-1}$

$$
GDP_i = M_i^p r_{iii}(\tilde{\phi}_i^p)
$$

A.4 Sales Distribution

We present the result for domestic firms selling domestically, but the expression is analog for the other activities.

$$
\begin{aligned}
prob(r_{iii}(\phi) > y) &= prob\left(E_i^1 \left(\frac{P_i\rho\phi}{w_i}\right)^{\sigma-1} > y \right) \\
&= prob\left(\phi > \left(\frac{y}{E_i^1}\right)^{\frac{w_i}{1-\sigma}} \frac{w_i}{P_i\rho} \right).
\end{aligned}
$$

where $r_i^m(\phi_i^m) = E_i^1(P_i\rho\phi_i^m)^{\sigma-1}$ is the revenue of a firm from country i with productivity equal to $\phi_{m,i}$ producing and selling domestically. Then $r_{iii}(\phi)$ is distributed Pareto with scale parameter r_i^m and shape parameter $\frac{\alpha}{\sigma-1}$. The distribution of sales is the truncation of the previous distribution. Then sales $r_{iii}(\phi)$ are distributed Pareto with scale parameter $r_{iii}(\phi^*)$ and shape parameter $\alpha/(\sigma-1)$, where $r_{iii}(\phi^*)$ are the sales of a firm with the cutoff productivity. As ϕ is distributed Pareto we can calculate this probability to be

$$prob(r_{iii}(\phi) > y) = \left(\frac{\phi_i^m}{\left(\frac{y}{E_i^1}\right)^{\frac{1}{1-\sigma}} \frac{w_i}{P_i\rho}} \right)^\alpha,$$

where $\phi_{m,i}$ is the scale parameter (the minimum value that ϕ can take) of the Pareto distribution. We can write the above expression as:

$$prob(r_{iii}(\phi) > y) = \left(\frac{(E_i^1)^{1/(\sigma-1)}(P_i\rho\phi_i^m/w_i)}{y^{1/(\sigma-1)}} \right)^\alpha$$

$$prob(r_{iii}(\phi) > y) = \left(\frac{E_i^1(P_i\rho\phi_i^m/w_i)^{(\sigma-1)}}{y} \right)^{\alpha/(\sigma-1)}$$

$$prob(r_{iii}(\phi) > y) = \left(\frac{r_i^m}{y} \right)^{\alpha/(\sigma-1)}$$

For the rest of activities we can operate in a similar way to obtain:

$$\text{Exporting firms} \Rightarrow Prob(r_{kii} > y) = \left(\left(\frac{P_k\rho\phi_i^m}{w_k\tau_{ki}} \right)^{(\sigma-1)} \frac{E_k^1}{y} \right)^{\alpha/(\sigma-1)}$$

$$\text{MP in country } k \Rightarrow Prob(r_{kii} > y) = \left(\left(\frac{P_k\rho\phi_i^m}{w_k\gamma_{ki}} \right)^{(\sigma-1)} \frac{E_k^1}{y} \right)^{\alpha/(\sigma-1)}$$

$$\text{BMP in } k \text{ to sell in } j \Rightarrow Prob(r_{kii} > y) = \left(\left(\frac{P_j\rho\phi_i^m}{w_k\tau_{jk}\gamma_{ki}} \right)^{(\sigma-1)} \frac{E_j^1}{y} \right)^{\alpha/(\sigma-1)}$$

$$(34)$$

where the numerator of each equation is the sales for each activity that correspond to the cutoff productivity level. As in the case of domestic sales, the equilibrium

distribution of sales for each activity is going to be Pareto with shape parameter $\alpha/(\sigma - 1)$ and scale parameter $r(\phi^*_{jki})$, where $r(\phi^*_{jki})$ is sales of a firm with the cutoff productivity level for a firm from country i producing in country k and selling to country j.

A.5   Algorithm to solve for the equilibrium

A.5.1   Verbal Explanation: Cutoffs

A firm will sell domestically if

$$\pi_{iii}(\phi) = \frac{E_i^1 P_i^{\sigma-1}}{\sigma} \left(\frac{\rho\phi}{w_i}\right)^{\sigma-1} - \kappa_i^d w_i \geq 0 \tag{35}$$

Profits will be equal to zero for the cutoff for domestic productivity, which we denote as ϕ^*_{iii}. All firms with productivities higher than ϕ^*_{iii} will sell domestically. Now, suppose a firm can also export to country k. Profits are given by:

$$\pi_{kii}(\phi) = \frac{E_k^1 P_k^{\sigma-1}}{\sigma} \left(\frac{\rho\phi}{w_i \tau_{ki}}\right)^{\sigma-1} - \kappa_i^x w_i \tag{36}$$

Setting this equation equal to zero, we can find the cutoff productivity ϕ^*_{kii} for a firm from country i exporting to country k. To fix ideas, let us keep aside the possibility of MP. Then, we have two possibilities for defining the exporting cutoffs

1. *Case A.* If all the exporting cutoffs are higher than the domestic cutoff in country i, that is if $\phi^*_{iii} < \phi^*_{kii} \ \forall k$, then the domestic and the exporting cutoffs are well calculated. Firms with productivities $\phi^*_{iii} < \phi < \phi^*_{kii}$ only sell in the domestic market, while firms with productivities $\phi > \phi^*_{kii}$ sell domestically and export.

2. *Case B.* If at least one exporting cutoff ϕ^*_{kii} is lower than the domestic cutoff ϕ^*_{iii}, then we need to re-calculate the cutoffs. Denote K_i^x the set of countries k for which the exporting cutoff (from country i to country k) is lower than the domestic cutoff. For countries $k \in K_i^x$ the exporting cutoff is equal to the domestic cutoff, $\phi^*_{iii} = \phi^*_{kii}$. The marginal firm entering in the domestic market (with productivity ϕ^*_{iii}) makes negative profits selling in the domestic market but these negative profits are compensated by the positive profits obtained by exporting to countries $k \in K_i^x$. Then, the productivity cutoff defined in the marginal entrant (ϕ^*_{iii}) solves the following equation:

$$\pi_{iii}(\phi^*_{iii}) + \sum_{k \in K^x} \pi_{kii}(\phi^*_{iii}) = 0. \tag{37}$$

Now consider the possibility for MP. Allowing for MP requires considering several cases for the way the domestic cutoff is defined. The profit for a firm from country i producing and selling in country k (performing DMP in country k) is given by:

$$\pi_{kki}(\phi) = \frac{E_k^1 P_k^{\sigma-1}}{\sigma} \left(\frac{\rho\phi}{w_k \gamma_{ki}} \right)^{\sigma-1} - \kappa_k^{MP} w_k \tag{38}$$

To fix ideas, let us ignore the possibility of exporting. We want to focus on how MP affects the calculation of the domestic cutoff. There are two cases again to consider:

1. *Case A.* If all the MP cutoffs are higher than the domestic cutoff in country i, that is if $\phi_{iii}^* < \phi_{kki}^*$ $\forall k$, then the domestic and the MP cutoffs are well calculated. Firms with productivities $\phi_{iii}^* < \phi < \phi_{kki}^*$ only sell in the domestic market, while firms with productivities $\phi > \phi_{kki}^*$ sell domestically and perform MP.

2. *Case B.* If at least one MP cutoff (ϕ_{kki}^*) is lower than the domestic cutoff, then we need to follow similar steps as in Case B. Denote by K_{ki}^{MP} the set of countries (k) for which the MP cutoff in country i (ϕ_{kki}^*) is lower than the domestic cutoff in country i (ϕ_{iii}^*). For countries $k \in K_{ki}^{MP}$ the MP cutoff is equal to the domestic cutoff $\phi_{kki}^* = \phi_{iii}^*$. The marginal firm entering into the domestic market (with productivity ϕ_{iii}^*) makes negative profits selling in the domestic market but these negative profits are compensated by the positive profits obtained by performing MP in countries $k \in K_{ki}^{MP}$. Then, the productivity of the marginal entrant in country i solves the following equation:

$$\pi_{iii}(\phi_{iii}^*) + \sum_{k \in K_{ki}^{MP}} \pi_{kki}(\phi_{iii}^*) = 0 \tag{39}$$

If we assume that firms can export and do MP, the procedure is the same. The only difference is that if we have exporting cutoffs and MP cutoffs that are below the domestic cutoff, then the productivity of the marginal entrant in country i solves the following equation:

$$\pi_{iii}(\phi_{iii}^*) + \sum_{k \in K^x} \pi_{kii}(\phi_{iii}^*) + \sum_{k \in K_{ki}^{MP}} \pi_{kki}(\phi_{iii}^*) = 0 \tag{40}$$

Finally, a firm may want to use a third country as an export platform (BMP). The profit for a firm from country i, producing in country k and selling in country j

is given by:

$$\pi_{jki}(\phi) = \frac{E_j^1 P_j^{\sigma-1}}{\sigma} \left(\frac{\rho\phi}{w_k \gamma_{ki} \tau_{jk}} \right)^{\sigma-1} - \kappa_k^x w_k \qquad (41)$$

Setting the above equation to zero, we can find the BMP cutoff productivity (ϕ_{jki}^*) for a firm from country i producing in country k and selling in country j. As in the previous cases we also need to consider two possibilities:

1. *Case A.* If all the BMP cutoff productivities for firms from country i producing in country k $(\phi_{jki}^* \ \forall j)$ are above the MP cutoff productivity for firms from country i producing in country k (ϕ_{kki}^*), then the BMP cutoffs are well calculated. Firms with productivities $\phi_{kki}^* < \phi < \phi_{jki}^*$ sell domestically and produce and sell in country k, while firms with productivities $\phi > \phi_{jki}^*$ sell domestically, produce and sell in country k and also do BMP from country k to country j.

2. *Case B.* If at least one BMP cutoff for firms from country i producing in country k $(\phi_{jki}^* \ \forall j)$ is below the MP cutoff productivity for firms from country i producing in country k (ϕ_{kki}^*), then we the MP cutoff, ϕ_{kki}^*, needs to be recalculated. Define J_{ki}^{BMP} the set of countries for which the BMP cutoff (ϕ_{jki}^*) is lower than the MP cutoff (ϕ_{kki}^*). Then the cutoff productivity for the marginal firm from country i performing MP in country k and BMP to country j solves:

$$\pi_{kki}(\phi_{kki}^*) + \sum_{j \in J_{ki}^{BMP}} \pi_{jki}(\phi_{kki}^*) = 0 \qquad (42)$$

As firms performing BMP have to pay the fixed cost of producing abroad (κ^{MP}) also, there will be no firm performing BMP and not MP, which implies that the equilibrium BMP cutoff is not going to be below the MP cutoff. After re-calculating the MP cutoff we have to check if the new MP cutoff is larger than the domestic cutoff. If it is larger, then the MP cutoff is well calculated, otherwise we need to re-calculate the domestic cutoff which will be the one that solves:

$$\pi_{iii}(\phi_{iii}^*) + \sum_{k \in K_{ki}^{MP}} \pi_{kki}(\phi_{iii}^*) + \sum_{k \neq i} \sum_{j \in J_{ki}^{BMP}} \pi_{jki}(\phi_{iii}^*) = 0 \,, \qquad (43)$$

A.5.2 Algorithm

In order to solve for the equilibrium we need $3N$ guesses. We start with N guesses for the $E_i^1 P_i^{\sigma-1}$, N guesses for wages (w_i) and N guesses for the mass of firms

in country i (M_i). With these guesses we can calculate the productivity cutoffs for each activity using Equations 35, 38, and 41. Once all cutoffs are computed we need to follow the next steps for each country. Take country i:

1. Check if the exporting cutoffs (ϕ^*_{jii}), MP cutoffs (ϕ^*_{kki}) and the BMP cutoffs (ϕ^*_{jki}) are well computed.

   (a) If all the cutoffs for country i producing in country k and selling to country j are bigger than the domestic cutoffs, then the domestic cutoffs are well computed. Proceed step 2.

   (b) If at least one cutoff is smaller than the domestic cutoff:

   - If the smallest cutoff is an exporting or an MP cutoff, then:

      i. Re-calculate the domestic cutoffs using equation 40.

      ii. Check that the new domestic cutoff is smaller than the rest of cutoffs (exporting, MP or BMP) or repeat the previous step incorporating the new smallest cutoff until there are no more cutoffs smaller than the domestic cutoff.

   - If the smallest cutoff is a BMP cutoff, then

      i. First re-calculate the new MP cutoff using Equation 42.

      ii. If this new MP cutoff is above the domestic cutoff, then check if there are no more cutoffs smaller than the domestic one. If this is the case, proceed to step 2.

      iii. If this new MP cutoff is smaller than the domestic cutoff, re-calculate the domestic cutoff using the following Equation

$$\pi_{iii}(\phi^*_{iii}) + \sum_{k \in K^x} \pi_{kii}(\phi^*_{iii}) + \sum_{k \in K^{MP}_{ki}} \pi_{kki}(\phi^*_{iii}) + \sum_{k \neq i} \sum_{j \in J^{BMP}_{ki}} \pi_{jki}(\phi^*_{iii}) = 0 \,,$$
(44)

   and repeat the process until there are no more cutoffs smaller than the domestic one,

2. Check that the MP cutoffs are well computed i.e. that all the BMP cutoffs are larger or equal than the MP cutoff in each case.

   (a) If all the BMP cutoff are above the MP cutoff, then the MP cutoff is well computed, and process is finished.

   (b) If at least one BMP cutoff is smaller than the MP cutoff, re-calculate the MP cutoff using equation 42.

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(c) Repeat the process until there are no more BMP cutoffs smaller than the MP cutoff

A.6   Calibration

Panel A

| | Exporting country | | | | |
Country	Argentina	Brazil	Chile	Uruguay	RW
Argentina	1	2.27	2.73	2.39	2.61
Brazil	1.48	1	2.36	1.76	2.03
Chile	1.66	2.07	1	2.27	1.93
Uruguay	1.75	2.19	2.57	1	2.68
Rest of the World	1.74	1.97	2.06	2.22	1

Panel B

| | Exporting country | | | | |
Country	France	UK	Italy	Netherlands	RW
France	1	1.82	1.62	1.74	1.61
UK	1.59	1	1.70	1.50	1.36
Italy	1.80	1.94	1	1.74	1.81
Netherlands	1.52	1.49	1.55	1	1.32
Rest of the World	1.81	1.80	1.78	1.77	1

Panel C

| | Country of origin | | | | |
Country	Argentina	Brazil	Chile	Uruguay	RW
Argentina	1	1.47	1.46	1.41	1.48
Brazil	3.75	1	3.08	2.45	2.49
Chile	2.49	2.35	1	2.15	1.81
Uruguay	–	–	–	1	2.02

Panel D

| | Country of origin | | | | |
Country	France	UK	Italy	Netherlands	RW
France	1	1.62	2.15	1.83	0.33
UK	1.65	1	2.20	1.68	0.28
Italy	1.40	1.49	1	1.55	0.29
Netherlands	1.65	1.47	–	1	0.29

Table 7: Calibration Results. Panels A, B: Iceberg Export Costs. Panels C, D: Efficiency coefficient of Multinationals (γ).